A

GENERAL TREATISE

OF

MENSURATION:

CONTAINING

Many useful and necessary Improvements.

Composed for the Benefit of

ARTIFICERS, BUILDERS, MEASURERS, SURVEYORS, GAUGERS, FARMERS, GENTLEMEN, YOUNG STUDENTS, &c.

The Whole being intended as an easy Introduction to several Parts of the

MATHEMATICKS.

By J. ROBERTSON, F. R. S.

The FOURTH EDITION.

LONDON:

Printed for J. NOURSE, in the Strand, Bookseller to His MAJESTY.

MDCCLXXIX.

TO

WILLIAM JONES, Esq;

FELLOW of the ROYAL SOCIETY.

SIR,

AS the publication of the former edition of this treatise, procured me the honour of your acquaintance, the consequences whereof have been one continued course of favours, which you have with the utmost generosity conferred on me; among these may be justly ranked the free access you have constantly allowed me to your books and papers, and also your friendly advice whenever desired: Therefore, no place more proper than in the present work, to return you my sincere thanks in this public manner.

AND, as many of the improvements in this impreſſion, are owing to your communications, ſo I beg you will accept of this ſmall offering, and grant it your protection; this I hope for, becauſe that candour and benevolence, which you ſo frequently, and on ſo many occaſions, exert to promote whatever appears deſigned for public, or for private uſe, is no leſs conſpicuous, than is your eminent knowledge in every branch of ſpeculative, and practical mathematics.

I am,

SIR,

Your moſt obliged,

And moſt humble Servant,

John Robertſon.

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PREFACE.

MENSURATION, if considered in its utmost extent, would include all the branches of practical mathematicks; and as these depend on the principles delivered in the speculative parts, therefore a Treatise *of* Mensuration *would not be an improper title to a system or course of mathematical sciences; for when these are applied to practical uses, there seem to be few things beside the measuring of lengths, superficies, solids, angles, forces, motion, duration and chance: But as custom has restrained the notion of mensuration to that of finding the lengths of lines, the superficial and solid content of figures; therefore the reader is to expect, in the following sheets, no more than what relates to the common acceptation of the word as now explained.*

About ten years ago, the author being desirous of putting together a few papers on Men-

ſuration *for the uſe of his pupils; to that purpoſe he peruſed the ſeveral treatiſes on that head; and conceiving them to be defective, either in matter or method, this induced him then to compoſe, what he thought, a new ſyſtem, diſpoſed in a quite different order from any of thoſe that were done before; which being approved of by ſeveral gooa judges, he publiſhed it; and the impreſſion being ſome time ſince ſold off, he reviſed the whole; made ſo many alterations and additions thereto, that he was in doubt whether to call it a ſecond edition, or a new work.*

This book is divided into three parts, and is preceded by an introduction, containing the doctrine of decimal fractions, and of duodecimal arithmetic; theſe are not here prefixed merely for their uſe in computing the ſuperficies and ſolidities of figures, for many other parts of the mathematicks have the ſame claim; but becauſe herein are ſeveral articles not very common, and which the young ſtudent might, perhaps, find quite neceſſary in reading the other parts of this treatiſe.

The firſt part contains the manner of computing the areas of right lined and circular plane figures.

The

The ſecond part contains rules for meaſuring the contents of ſolids comprehended under right lined and circular figures.

The third part ſhews how the ſuperficial contents of thoſe figures commonly called conic ſections are to be computed; and alſo, the ſurfaces and contents of ſeveral ſolids generated by the motion of ſuch figures about certain right lines: To which are added, many things relating to the ſubject of Menſuration, *the particulars of which are enumerated in the contents.*

The whole is illuſtrated with a great variety of, what is apprehended to be, uſeful examples; which are ſo contrived, as to ſerve for exerciſes to ſeveral of the preceding propoſitions, and adapted to ſuch uſes as often occur in the common affairs of life: In thoſe which relate to artificers works concerned in building, the operations are generally performed, both decimally and duodecimally: Alſo, the cuſtoms and allowances in the works of ſuch artificers, are inſerted in their proper places.

The reader will here find, not only what is to be met with in other books on the ſame ſubject, and what are diſperſed in miſcellaneous works,

works, but many things, perhaps, entirely new; and others (not very common) are so disposed as to be much more practicable and useful, than they appeared to be, in the form originally given them.

As this book is chiefly intended for such persons as are employed in practical business, therefore the demonstrations of the rules are omitted; referring more inquisitive readers to the elementary books of Geometry: *Nevertheless, the learner will here meet with an easy introduction to several parts of the mathematics; and those who have made farther advances, will, at least, find in this, a common-place-book for many rules, which may not be of sufficient importance to burthen their memory with.*

Upon the whole, the author has spared no pains to make it generally useful, and therefore, if any oversight has escaped him, either in the press or otherwise, among such a multitude of articles, he hopes the candid reader will generously excuse him; his view being never to find fault with others, but of endeavouring, as much as in him lies, to promote these Studies, so highly beneficial to mankind.

October 29, 1747.

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VII.

THE

INTRODUCTION.

Containing the

DOCTRINE

OF

DECIMAL FRACTIONS.

SECTION I.

ANY thing, considered as one, is called an *unit*. *As one yard, one pound, one gallon, &c. are all units, of their respective kinds.*

But one yard, one pound, one gallon, &c. may each be considered as compos'd of several lesser quantities; then are these component things, considered as *parts* of their respective units.

 Thus

Thus one yard may be conceived to consist of 36 inches, or of 3 feet, or of four quarters; then are inches, feet and quarters, taken as parts of a yard, &c.

2. A fraction is some part or parts of an *unit*. Fractions arise from Division, and are expressed by writing the Divisor under the Remainder, with a Line drawn between them.

Thus if 38 *was to be divided by* 6, *the quotient would be less than* 7, *and more than* 6; *that is, it would be* $6\frac{2}{6}$.

3. In a fraction thus constituted, the number below the line is called the *denominator*, and shews into how many parts the unit is supposed to be divided: The number above the line is called the *numerator*, and shews the value of the fraction in parts of the denominator: Expressions of this kind are called *vulgar fractions*.

When the denominators of vulgar fractions consist of several digits, their management is attended with some trouble; to avoid which, the following method will chiefly be used in this Treatise.

4. Fractions, whose denominators consist of unity with one, or more cyphers, are called *decimals, or decimal fractions*.

5. Decimal fractions are commonly wrote without their denominators; these being always understood to consist of an unit, with as many cyphers annexed (on the right hand) as the decimal fraction (or numerator) has places: and to distinguish integral places from fractional ones, the latter have a point, or comma, set before the left-hand place.

Thus

Thus 0,6 is underſtood to be $\frac{6}{10}$;
And 0,47 - - - - $\frac{47}{100}$;
And 0,763218 - - - $\frac{763218}{1000000}$;
And 246,385 - - - 246 $\frac{385}{1000}$
&c.

6. A finite decimal, is that which ends at a certain number of places; but an infinite, that which no where ends.

7. A *recurring*, or *circulating decimal*, is that wherein one or more figures are continually repeated.

Thus 42,387666, &c. *is call'd a ſingle circulate or recurring decimal.*

And 285,264264264, &c. *is call'd a compound recurring decimal.*

8. The firſt place next the mark of diſtinction in any decimal expreſſion, is called the place of primes; and the following places are called ſeconds, thirds, fourths, *&c.*

9. Cyphers to the right-hand of decimals, neither increaſe nor decreaſe their value; but cyphers between the ſeparating point and the digits of the decimal, diminiſh the value.

Thus 0,5, is $\frac{5}{10}$.
But 0,05, is $\frac{5}{100}$.
And 0,005, is $\frac{5}{1000}$, *&c.*

The like is to be underſtood in other decimals.

10. In any mixed or fractional number, if the mark of diſtinction be removed one, two, three, *&c.* places to the right-hand, then every place in that number will be 10,100, 1000, *&c.* times greater than it was before. But if the mark be removed towards the left-hand, then every place will be diminiſhed in the ſame manner.

Thus	And
357,984	357,948
3579,84	35,7948
35798,4	3,57948
&c. increasing	&c. decreasing.

A number consisting of integral and fractional places, is called a mixed number.

11. Every finite decimal may be considered as infinite, by making cyphers to recur. For they do not alter the value of the decimal.

12. Any decimal expression may be continued at pleasure, by repeating the circulating figure or figures.

13. In all operations, if the result consists of several nines, reject them, and make the next superior place an unit more; thus for 7,23999, &c. write 7,24.

14. In all circulating numbers, dash the first and last of the recurring digits, omitting the intermediate places; thus, 4,283 or 7,343343343, &c.

SECTION II.

REDUCTION.

OR the methods used to bring any vulgar fraction, or an expression of different denominations to its equivalent decimal value: Or any decimal expressions, to its value in different denominations.

CASE I.

15. *To reduce a vulgar fraction, to its equivalent decimal one*

RULE.

RULE.

Divide the numerator (with as many cyphers annexed, as may be neceſſary,) by the denominator; and the quotient will be the decimal ſought. Obſerving, that for every cypher uſed with the numerator, there muſt be a cypher, or digit, in the decimal expreſſion found in the quotient; and the comma, or mark of diſtinction, muſt be ſet on the left-hand thereof.

EXAMPLES.

I. $\frac{1}{4}$ is equal to 0,25;
II. $\frac{1}{2}$ - - - 0,5
III. $\frac{3}{4}$ - - - 0,75
IV. $\frac{5}{8}$ - - - 0,625
V. $\frac{7}{12}$ - - - 0,583

From the laſt example, it will be eaſy to conceive how circulates are generated; and alſo, to ſee that 3 would continually repeat.

What is the decimal fraction equal to $\frac{9}{56}$?

```
56)9,00000000(0,16071428, &c.
   340
    400
     80
     240
      100
      480
       32
```

VII. *What is the decimal fraction equal to* $\frac{17}{286}$?

286) 17,0000000 (0,0594405, &c.

2700

1260

1160

1600

170

VIII. *What is the decimal fraction equal to* $\frac{217}{2495}$?

2495) 217,00 (0,8697394, &c.

17400

24300

18450

9850

23650

11950

1970

IX. *What is the decimal fraction equal to* $\frac{83}{9768}$?

9768) 83,000 (0,008497133, &c.

48560

94880

69680

13040

32720

34160

4856

In

In Example VII. may be seen the generation of a compound recurring decimal: For after 7 cyphers are used, the remainder 170, with another cypher annexed, is equal to the number began with; consequently the same figures, *viz.* 594405, will repeat in the quotient; and after them, the same again, *&c.* and this is distinguished by dashing the first and last figures.

And the like in the 9th example above.

There is seldom a necessity of obtaining more than 6 places in the decimal or quotient, these being sufficiently exact for most uses.

It may be observed, in each of the three last Examples, that one or more of the first places of the quotient, are possessed by cyphers; and this is, because two or more cyphers are used in the dividend before a digit arises in the quotient.

CASE II.

16. *To reduce the different denominations of Money, Weights, Measures, &c. to their equivalent decimal values.*

RULE.

Write the given denominations, or parts, orderly under each other; the inferior or least parts, being uppermost: Let these be dividends.

Against each part, on the left-hand, write the number thereof, contained in one of its next superior: Let these be divisors.

Then, beginning with the upper one, write the quotient of each division, as decimal parts, on the right-hand of the dividend next below it; and let this mixed number be divided by its divisor, *&c.*

And the last quotient will be the decimal sought.

EXAMPLES.

I. *Reduce* 10s. 8d. *to its equivalent decimal of a* pound sterling.

12	8
20	10,6
	0,53

Therefore 0,58l. *is equal to* 10s. 8d.

II. *What decimal of* 1l. *is equivalent to* 13s. 10½d.?

4	2
12	10,5
20	13,875
	0,69375l.

III. *What decimal of* 1l. *is equal to* 15s. 9¾d.?

4	3
12	9,75
20	15,8125
	0,790625l.

IV. *What decimal of* 1l. *is equal to* 19s. 11¼d.?

4	1
12	11,25
20	19,9375
	0,996875l.

V. *What decimal part of a pound* troy, *is equivalent to* 10 *oz.* 18 *dwts.* 16 *grs.*?

24 {	4	16
	6	(4
20		18,6
12		10,93
		0,91 ℔ troy.

VI. *What decimal part of a C. wt. is equivalent to* 3 *qrs.* 16 ℔ 12 *oz.* averdupoise.

16 {	4	12
	4	(3
28 {	4	16,75
	7	(4, 1875
4		3,598214, &c.
		0,899553 C. w.

VII. *What decimal part of a foot is equal to* 10 *in.* 9 *pts.* 7 *sec.*

12	7
12	9,583
12	10,79861
	0,899884 f.

VIII. *What decimal part of a degree of a circle, is equal to* 48 *min.* 37 *sec.* 54 *thirds.*

60	54
60	37,9
60	48,6316
	0,810527 deg.

17. *Note,*

17. *Note*, For sterling money, the decimal may be wrote in one line, by the following

RULE.

Write half of the greatest even number, in the given shillings, for the place of primes.

Let the farthings, in the given pence and farthings, possess the places of the seconds and thirds. Observing, if the given shillings are odd, to increase the place of seconds by 5.

And to increase the thirds by as many units as there are times 24 in the pence and farthings.

Divide half the number of farthings, in the pence and farthings (rejecting 24, or sixpence, if there is one) by 12, the quotient written after the three places before found, will give the decimal required.

EXAMPLES.

I.	10 *s.* 8 *d.*	is equal to	0,533 *l.*
II.	13 *s.* $10\frac{1}{2}$ *d.*	———	0,69375 *l.*
III.	15 *s.* $9\frac{3}{4}$ *d.*	———	0,790625 *l.*
IV.	19 *s.* $11\frac{1}{4}$ *d.*	———	0,996875 *l.*
V.	1 *s.* $10\frac{1}{4}$ *d.*	———	0,0927083 *l.*
VI.	0 *s.* $8\frac{3}{4}$ *d.*	———	0,0364583 *l.*
VII.	0 *s.* $2\frac{1}{2}$ *d.*	———	0,010416 *l.*
VIII.	0 *s.* $0\frac{3}{4}$ *d.*	———	0,003125 *l.*

One of these examples explained will make the rule familiar.

In the V. *viz.* 1 *s.* $10\frac{1}{4}$ *d.* half of 1 *s.* is 0, write 0 in the place of primes: $10\frac{1}{4}$ *d.* is 41 farthings; and 1 added (for the 24 contained in 41,) makes 42; and 5,0 added (for the odd shilling,) makes 92; therefore the three first places of the decimal, are 0,092: now 24 taken from 41, leaves

 17;

17; its half is 8,5; which divided by 12, gives 7803; these wrote, as they arise, after the former three places, make 0,0927803 for the decimal required.

CASE III.

18. *A decimal fraction being given; to find its equivalent value, in inferior denominations.*

RULE.

Multiply the given decimal, by the number of parts in the next lesser denomination; from the product, cut off as many places to the right-hand, as there are in the given decimal.

Multiply these by the parts in the next lesser denomination, and from this product cut off as before.

And thus proceed until the least denomination is arrived at, then the several parts cut off on the left-hand, are equivalent to the given decimal.

EXAMPLES.

I. *What is the value of* 0,72896 *l.?*

0,72896
20
14,57920
12
6,95040
4
3,80160

Answer, 14 *s.* 6$\frac{3}{4}$ *d.*

II. *What is the value of* 0,92384 *l.?*

0,92384
20
18,47680
12
5,72160
4
2,88640

Answer, 18 *s.* 5$\frac{1}{2}$ *d.*

III.

III. *What is the value of 0,798645 of C. wt. averdupoiſe?*

0,798645
4 *Quarters.*
3,194580
28 *Pounds.*
1556640
389160
5,448240
16 *Ounces.*
7,171840

Anſwer 3 *qrs.* 5 ℔ 7 *oz.*

IV. *What is the value of 0,87628 of a ℔ troy?*

0,87628
12 *Ounces.*
10,51536
20 *Pennyweights.*
10,30720
24 *Grains.*
122880
61440
7,37280

Anſw. 10 *oz.* 10 *dwt.* 7*gr.*

19. But the value of the decimal part of a pound ſterling may be expreſſed in one line; thus.

Double the place of primes for ſhillings, and if the ſecond place be 5, or exceed 5, reckon one ſhilling more: the figures in the ſecond and third places [rejecting 5 in the ſecond place] are ſo many farthings, abating one for every 24.

EXAMPLES.

I.	The value of 0,92763 *l.*	is	18 *s.*	$6\frac{1}{2}$ *d.*
II.	- - - - 0,87638 *l.*	is	17 *s.*	$6\frac{1}{2}$ *d.*
III.	- - - - 0,09937 *l.*	is	1 *s.*	$11\frac{3}{4}$ *d.*
IV.	- - - - 0,0428 *l.*	is		$10\frac{1}{4}$ *d.*
V.	- - - - 0,0095 *l.*	is		$2\frac{1}{4}$ *d.*

SECTION III.

ADDITION and SUBSTRACTION.

CASE I.

20. *To add or subſtract finite Decimals.*

RULE.

ADD, or ſubſtract, as in whole numbers, and from the ſum, or difference, cut off as many decimal places as are the greateſt number of decimals in any of the given expreſſions: but obſerve, that the ſeparating commas in each expreſſion, be placed directly underneath each other; for then units, *&c.* (if any) will fall under units, *&c.* and primes, ſeconds, thirds, *&c.* under primes, ſeconds, thirds, *&c.*

EXAMPLES in ADDITION.

347,256	3468,04973	8267,4031
5,61739	24,3675	8,965
0,1725	148,952	453,720853
43,	37,284695	76,98124
36,5	5,125	0,2738107
432,54589	3683,778925	8807,3440037

EXAMPLES in SUBSTRACTION.

From	384,76215	Minuend.	From	426,8
Take	86,2095	Subtrahend.	Take	379,604832
	298,55265	Differ. or Rem.		47,195168

CASE II.

21. *To add decimals, wherein there are single recurring figures.*

RULE.

Make every line end at the same place, by filling up the vacancies with the recurring digits, and annexing a cypher or cyphers, to the finite terms: then add as before; only increase the sum of the right-hand row, with as many units as it contains nines, and the figure in the sum under that place will be a circulate.

EXAMPLES.

8439,6548	5391,357	217,849$\dot{6}$	876,29$\dot{3}$
281,04$\dot{6}$	72,3$\dot{8}$	42,17$\dot{6}$	5,8764289
7042,3$\dot{5}$	187,2$\dot{1}$	,52$\dot{3}$	,0358$\dot{6}$
9,83	4,296$\dot{8}$	58,30048	628,4593$\dot{8}$

In each of these examples there are single recurring figures, which before they are added, must be made to end together, and then they will stand as follows:

8439,6548$\dot{0}$	5391,457$\dot{0}$	217,849$\dot{6}$66
281,04$\dot{6}$66	72,3$\dot{8}$88	42,17$\dot{6}$666
7042,3$\dot{5}$555	187,2$\dot{1}$11	,52$\dot{3}$333
9,83$\dot{7}$77	4,296$\dot{8}$	58,30048$\dot{0}$
15772,8948$\dot{0}$	5655,253$\dot{5}$	318,85014$\dot{6}$

876,29$\dot{3}$33333
5,8764289$\dot{0}$
,0358$\dot{6}$666
628,4593$\dot{8}$888
―――――
1510,6650177$\dot{8}$

Here

Here it may be obſerved, that in each example the circulates are carried one place farther than the finite expreſſions, and to the ſum of that row, there are as many units added, as there were nines in the ſum.

CASE III.

22. *To find the difference between two decimal fractions with ſingle circulates.*

RULE.

Make both end together as in addition: and if the right-hand figure of the ſubtrahend (being a circulate) be bigger than the figure over it in the minuend, inſtead of borrowing 10, as in ſubſtraction of whole numbers or finites; borrow 9 in this place, the reſt as uſual, and the right-hand place of the remainder will be a circulate.

From	$476,3\dot{2}$	$289,576$	$325,791\dot{8}$	$643,9207$
Take	$84,769\dot{1}$	$92,5846$	$37,095$	$583,7\dot{6}$

Theſe examples being made to end together, as before directed, will ſtand thus:

$476,3\dot{2}22$	$289,576\dot{0}$	$325,791\dot{8}$	$643,9207\dot{0}$
$84,769\dot{1}$	$92,584\dot{6}$	$37,095\dot{0}$	$583,7\dot{6}666$
$391,552\dot{4}$	$196,991\dot{3}$	$288,696\dot{8}$	$60,1540\dot{3}$

SECTION IV.

MULTIPLICATION.

CASE I.

23. *When both factors are finite decimals.*

RULE.

PLACE the factors, and multiply them as in whole numbers; and from the product, towards the right-hand, cut off as many places for decimals, as there are fractional parts in both factors together.

But if it so happen, that there are not so many places in the product, supply the defect with cyphers to the left-hand.

3684,7928	,2365	,0347
84,216	,2435	,0236
221087568	11825	2082
36847928	7095	1041
73695856	9460	694
147391712	4730	,00081892
294783424	,05758775	
310318,5104448		

In the first example, there being four decimal places in the multiplicand, and three in the multiplier, which together are seven; therefore cut off seven figures from the right-hand of the product for decimals; those to the left-hand being integers.

In

In each of the second and third examples, the decimal places in both factors are eight, but there arises in the second example, only seven figures in the product, and in the third only five; therefore in the second example, annex one cypher, and in the third, three cyphers to supply the defect.

CASE II.

24. *If the right-hand figure of the multiplicand be a circulate.*

RULE.

Multiply the multiplicand as before, by every figure in the multiplier; observing to increase the right-hand figure of each resulting line, by as many units as there are nines in the first product of that line; and the right-hand figure of each line will be a circulate; therefore in the adding the several lines together, make them end at the same place, as shewn in addition.

EXAMPLES.

First.

$$\begin{array}{r} 821,734\dot{8} \\ ,8 \\ \hline 657,3876\dot{4} \end{array}$$

Second.

$$\begin{array}{r} 3756,27\dot{3} \\ 4,6 \\ \hline 2277764\dot{0} \\ 151850933\dot{3} \\ \hline 17462,857\dot{3} \end{array}$$

Third.

$$\begin{array}{r} 8946,8370\dot{6} \\ 48,57 \\ \hline 626278594\dot{6} \\ 447341853\dot{3}3 \\ 71574696\dot{5}333 \\ 357873482\dot{0}666 \\ \hline 434547,876328\dot{0} \end{array}$$

CASE

CASE III.

25. *When the multiplier is a single circulate.*

RULE.

Multiply by it, as tho' it was a finite digit, setting the product one place forwarder than ordinary, towards the left-hand; divide the result by nine, continuing the quotient (if needful) till it arrives at a circulate; then beginning at the place under the right-hand figure of the multiplicand, cut off for fractions as before, and this will be the true Product.

EXAMPLES.

First.

```
     438,6297
           ,6
  -----------
  9)263177820
  -----------
   292,41980
```

Second.

```
    5820,39462
           ,37
  ------------
  9)4074276234
  ------------
    4526973593
   17461118386
  ------------
   2198,815745 3
```

Third.

```
      47,63
      2,848
  ---------
  9)  23815
  ---------
      26461
     19052
    38104
    9526
  ---------
  135,53381
```

```
       2,848
       47,63
     -------
        8536
      170733
     1991888
    11387222
   ---------
   135,53381
```

This

This third Example is wrought by Case II. and III. and the results are exactly the same.

In this case, if there are any other figures in the multiplier, beside the circulate, multiply by them like finite digits.

CASE IV.

26. *When the multiplicand and multiplier are each a single circulate.*

RULE.

In multiplying the multiplicand by each figure in the multiplier, observe the directions given in Case II. but the first line (or that line produced by multiplying the multiplicand, by the circulate in the multiplier) must be managed as directed in Case III.

EXAMPLES.

$$\begin{array}{r} 463{,}970\dot{4} \\ 8{,}64\dot{7} \\ \hline 9)\;927940\dot{8} \\ \hline 10310454,\ \&c. \\ 1855881\dot{7}7 \\ 2783822\dot{6}66 \\ 3711763\dot{5}555 \\ \hline 4009{,}73568\dot{5}4 \\ \hline \end{array}$$

$$\begin{array}{r} 862357{,}9\dot{3} \\ 36{,}1\dot{6} \\ \hline 9)\;51741475\dot{3} \\ \hline 5749052\dot{8}1 \\ 8623579\dot{7}2 \\ 43117896\dot{1}1 \\ 25870737\dot{6}666 \\ \hline 30326253{,}59\dot{8}1 \\ \hline \end{array}$$

$$\begin{array}{r} 0{,}539276\dot{5} \\ 935{,}\dot{7} \\ \hline 9)\;3774935\dot{8} \\ \hline 419437\dot{8}2,\ \&c. \\ 1617829\dot{6}6 \\ 2696382\dot{7}77 \\ 4853489\dot{0} \\ \hline 514{,}349994\dot{7}6 \\ \hline \end{array}$$

By

By dividing by 9 in these examples, there results in each a compound circulate for the first line; in the other lines the circulates are single; now each of them being filled up, (by Case II. in Addition) till they end under the right-hand place of the multiplier, they are added up like whole numbers, but the sum of the right-hand row is increased by as many units as are tens in the sum of that row where the compound circulate begins; and the figure in the sum under that row, is the first figure of a compound circulate; the other figures of it are found, by continuing the lines to end with the last figure of the compound circulate in the first line.

CASE V.

28. *Two decimal factors being given, to reserve in their product any number of places.*

RULE.

Under that place in the multiplicand, thought necessary to be retained in the product, write the units place of the multiplier, and invert the order of all its other places; that is, write the decimals on the left, and the integers (if any) on the right.

In multiplying, omit those places in the multiplicand, to the right of the digit multiplying by, and let the right-hand place of every line stand under each other.

In each line, let the lowest place be increased by the carriage, which would arise from the omitted places; that is, carrying 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35, *&c.* instead of carrying 1 for every 10, *&c.*; and the Sum of these lines will give the product generally exact.

This Rule is of use, to contract the work that would arise in multiplying with many decimal places, by omitting the superfluous ones.

EX-

EXAMPLES.

Multiply 384,672158 by 36,8345.

Now seeing there would be 10 decimal places in the product, whereof the greatest part are unnecessary; therefore keep only four decimal places in the product.

384,672158	Multiplicand.	384,672158	
5438,63	Multiplier inverted.	36,8345	
115401647	.	1923	360790
23080329	..	15386	88632
3077377	...	115401	6474
115402	....	3077377	264
15387	.....	23080329	48
1923	......	115401647	4
14169,2065	......	14169,2066	038510

Here the Example is wrought both ways, by which may be easily seen what is saved by the last Rule.

In this Example, because it is intended to keep 4 decimal places in the product, set 6, the unit's place of the multiplier under 1, the *4th* place in decimals of the multiplicand, and invert the order of all the rest of the figures: Then say three times 8 is 24, and carry 2; 3 times 5 is 15, and 2 is 17, now set down the 7 and carry 1, *&c.* because this is the product arising by multiplying the 5 that stands over the 3.

Again 6 times 8 is 48, and carry 5; 6 times 5 is 30, and 5 is 35, and carry 3; 6 times 1 is 6, and 3 is 9. Now being come to the figure over the 6, set down 9, *&c.*

Again,

Again, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17; now being come to the figure over the 8, set down 7, and carry 1, &c. Proceeding in like manner with every figure in the inverted multiplier, till all is done.

Multiply 3,141592 by 52,7438, and reserve 4 decimal places in the product.

```
 3,141593
 8347,25
 --------
 1570796
   62832
   21991
    1257
      94
      25
 --------
 165,6995
```

Multiply 257,356 by 76,48, and to have the product only in whole numbers.

```
 257,356              257,356
 84,67                  76,48
 -------      ---------------
 18015           20|58848
  1544          102|9424
   103         1544|136
    20        18014|92
 -------      ---------------
 19682        19682,58688
```

SEC-

SECTION V.

DIVISION.

CASE I.

29. *When the divisor and dividend are finite decimals.*

RULE.

DIVIDE as in integers, and from the right-hand of the quotient, point off for decimals, so many places, as the decimal places used in the dividend, exceed those of the divisor; and those to the left, if any, are integers.

If the places in the quotient, are not as many as this Rule requires, supply the defect with cyphers, on the left-hand.

But if the decimal places in the divisor, be more than those in the dividend, add cyphers as decimals to the dividend, till the number of decimal places in the dividend, is, at least, equal to those in the divisor, and the quotient will be integers until all these cyphers are used.

```
43,6) 3424,6056 (78,546
      ---------
       3726
       ----
        2380
        ----
         2005
         ----
          2616
          ----
```

c,675)

```
0,675) 3877875,000 (5745000
       5028
        3037
         3375
           000

0,347) ,0081892 (,0236
          1249
           2082
```

If these examples be compared with the foregoing Rule, it will be very easy to see how they are performed, and the quotient rightly adjusted.

Or, the place of the first digit in the quotient, will always be equal to that place of the dividend, under which, falls the units of the divisor, when multiplied by that quotient digit.

Thus in Ex. I. the divisor 43,6, multiplied by 7, the first digit in the quotient; the place of units in the product, will fall under the place of *tens* in the dividend; therefore the place of the first digit, 7, in the quotient, will be that of *tens*.

In Ex. II. the place of units in the divisor, falls under the place of *millions* in the dividend; therefore 5, the first digit in the quotient, will be in the place of *millions*.

In Ex. III. the place of units in the divisor, falls under that of seconds in the dividend; therefore 2, the first digit in the quotient, will possess the place of seconds; and consequently a cypher will be in the place of primes.

CASE

CASE II.

30. *If the dividend be a circulate.*

RULE.

To the remainders, bring down the recurring figure, until the quotient is as exact as required.

4,72) 8,33 (1,765536, &c.
3613
3693
2613
2533
1733
317, &c.

54,283)0,876527 (,016147, &c.
333697
79997
257147
400157
20176, &c.

CASE III.

31. *If the divisor be a recurring decimal.*

RULE.

Write the divisor and dividend in the order of division; and under these, write them a second time; but each removed as many places to the right,

right, as are the number of circulating places in the divisor; then substracting the lower line from the upper one, let the remainder be the divisor and dividend; and the quotient of these will be the true one.

EXAMPLES.

```
First.  8946,83706) 434547,876328 (48,57
         894,68370   43454,7876328
        8052,15336) 391093,0886952
                      6900695429
                       4589727415
                        5636507352
                        0000000000
```

It will be easy to see how this Example is performed, by observing the rule; and by comparing it with Example III. to Case I. in multiplication, may be seen how these two rules prove each other.

```
Second.      748,64) 47,464057 (,0634
              74,86   4,746405
             673,78) 42,717652
                        229085
                         269512
                              0
```

```
Third.       2,845) 135,53381 (47,63
               284   13,55338
             2,561) 121,98043
                       19540
                        16134
                         7683
                            0
```

Fourth.	862357,9$\cancel{7}$)	30326253,59$\cancel{8}$1$\cancel{4}$ *see Ex. p.* 18.
	86235,79	3032625,35981
	776122,13	27293628,238$\cancel{3}$3 (35,1$\cancel{6}$, &c.
		400996433
		129353688
		517414753
		51741475, &c.
Fifth.	76,4$\cancel{7}$)	8293,764$\cancel{3}$ (108,44
	7,64	829,3764
	68,83)	7464,3879
		58138
		30747
		32159
		4627

CASE IV.

32. *To contract the work of division, when the divisor consists of many decimal places.*

RULE.

Let each remainder be a new dividend, and for each such new dividend, point off one figure from the right hand of the divisor; observing at each multiplication to have regard to the increase of the figures so cut off as in contracted multiplication.

EXAMPLES.

```
384,672158) 14169,2066239510 (36,8345
 .....       11540,16474
             -----------
              262904188 .
              230803295 .
              ---------
               32100893 ..
               30773772 ..
               --------
                1327121 ...
                1154016 ...
                -------
                 173105 ....
                 153869 ....
                 ------
                  19236 .....
                  19234 .....
                  -----

9,365407) 87,076326 (9,297655
 ......   84,288663
          ---------
           2787663 .
           1873081 .
           -------
            914582 ..
            842886 ..
            ------
             71696 ...
             65558 ...
             -----
              6138 ....
              5619 ....
              ----
               519 .....
               468 .....
               ---
                51 .....
                47 ......
                --
                 4 ......
```

This will not be difficult if it be carefully examined.

SECTION VI.

33. *A decimal fraction being given, to find its least equivalent vulgar one.*

CASE I.

If the decimal be finite.

RULE.

UNDER the given decimal, write its proper denominator; then the terms of this fraction, divided by their greatest common measure, will give the least equivalent vulgar fraction required.

Note, The greatest common measure of two given numbers, is found by dividing the greater by the lesser, the lesser by the remainder, &c. always dividing the last divisor by the last remainder, until nothing remains; and the last divisor is the greatest common measure.

EXAMPLES.

I. Required the least vulgar fraction equivalent to the decimal 0,5?

Then $0,5 =^{*} \frac{5}{10} = \frac{1}{2}$.

II. What is the least vulgar fraction to 0,75?

Now $0,75 = \frac{75}{100} = \frac{3}{4}$.

III. What is the least vulgar fraction to 0,6?

Now $0,6 = \frac{6}{10} = \frac{3}{5}$.

* For the explanation of the signs, see Sect. XI.

IV. What

IV. What is the least vulgar fraction to 0,625?

Now $0,625 = \frac{625}{1000} = \frac{5}{8}$.

For 625)1000(1
375)625(1
250)375(1
125)250(2
0

And $125)\frac{625}{1000}(\frac{5}{8}$.

V. What is the least vulgar fraction to 0,5625?

Now $0,5625 = \frac{5625}{10000} = \frac{9}{16}$.

For 5625)10000(1
4375)5625(1
1250)4375(3
625)1250(3
0

And $625)\frac{5625}{10000}(\frac{9}{16}$.

CASE II.

34. *If the given decimal be a recurring one.*

RULE.

Make the given decimal, the numerator of a vulgar fraction, whose denominator shall consist of as many nines, as there are recurring places in the given decimal; the terms of this fraction, divided by their greatest common measure, will give the least equivalent vulgar fraction required.

If one or more of the left-hand places in the given decimal be cyphers, annex as many cyphers to the right-hand of the nines in the denominator.

EXAMPLES.

I. Required the least equivalent vulgar fraction, to $0,\dot{3}$?

Now $0,\dot{3} = \frac{3}{9} = \frac{1}{3}$.

II. What is the least vulgar fraction to $0,\dot{2}5\dot{7}$?

Now $0,\dot{2}5\dot{7} = \frac{257}{999}$.

III What is the least vulgar fraction to $0,0\dot{5}9440\dot{5}$?

Now $0,0\dot{5}9440\dot{5} = \frac{594405}{9999990} = \frac{17}{286}$.

For the greatest common measure to $\frac{594405}{9999990}$ is 34965.

And $34965)\frac{594405}{9999990}(\frac{17}{286}$

(See Exam. VII. P. 6.)

CASE III.

35. *When part of the given decimal is finite, and part circulates.*

RULE.

1. To the right-hand of as many nines, as there are recurring places, annex as many cyphers as there are finite places; and let this be the denominator.
2. Multiply the nines in the denominator, by the finite part; to the product, add the recurring part, and make this the numerator.

3. The

3. The terms of this vulgar fraction divided by their greatest common measure, will give the least equivalent fraction required.

Note, The first and second Cases, are included in this.

EXAMPLES.

I. Required the least equivalent vulgar fraction to the decimal $0,5\dot{3}$?

Now $0,5\dot{3} = \frac{48}{0} = \frac{8}{15}$.

For 90 is the denominator,

And $\overline{9 \times 5} + 3 = 48$ is the numerator.

II. What is the least vulgar fraction to $0,58\dot{3}$?

Now $0,58\dot{3} = \frac{525}{900} = \frac{7}{12}$.

For $0,58\dot{3} = \frac{58}{100} + \frac{3}{900}$; And $\overline{58 \times 9} + 3 = 525$.

And the greatest common measure is 75.

Then $75)\frac{525}{900}(\frac{7}{12}$.

III. What is the least vul. fraction to $0,008\dot{4}9713\dot{3}$?

Now $0,008\dot{4}9713\dot{3} = \frac{8497125}{999999000} = \frac{83}{9763}$.

For $0,008\dot{4}9713\dot{3} = \frac{8}{1000} + \frac{497133}{999999000}$.

And $\overline{8 \times 999999} + 497133 = 8497125$.

And the greatest common measure is 102375.

Then $102375)\frac{8497125}{999999000}(\frac{83}{9763}$.

See Example IX. P. 6.

36. The three Cases may be solved by the following

RULE.

1. Under the given decimal set its proper denominator.
2. Write this vulgar fraction under itself; but remov'd as many places forward to the right-hand, as there are recurring places.
3. Subtract the under numerator from the upper one, and the under denominator from the upper one; then will the remainders constitute a vulgar fraction equivalent to the given decimal.

Reduce this to its least terms as before.

EXAMPLES.

I. To find a vulgar fraction to $0,58\dot{3}$.

From $\frac{583}{1000}$

Take $\frac{58}{100}$

Remains $\frac{525}{900} = 0,58\dot{3}$; as before.

II. Required the vulgar fraction to $0,008\dot{4}9713\dot{3}$.

From $\frac{008497133}{1000000000}$

Take $\frac{008}{000}$

Leaves $\frac{8497125}{999999000} = 0,008\dot{4}9713\dot{3}$ as before.

SCHO-

SCHOLIUM.

37. The truth of these Rules, and indeed of all the Rules concerning recurring decimals, may be easily examined, if it be considered, that every circulating or recurring decimal, is a geometrical series infinitely decreasing to 0.

Thus $0{,}\dot{3}$ &c. $= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$, &c.

And

$0{,}\dot{2}5\dot{7}$, &c. $= \frac{257}{1000} + \frac{257}{1000000} + \frac{257}{1000000000}$, &c.

And the sum of such a decreasing series, is equal to the square of the first term, divided by the difference between the first and second terms.

SECTION VII.

Of TABLES.

IN decimal computations, 'tis of use to have Tables of the decimal values of the parts of coin, weight, measures and time; therefore the following Tables, and the manner of constructing them, are here introduced.

38. CONSTRUCTION *of* TABLE I.

I. The shillings 19, 18, 17, 16, &c. are separately divided by 20, and the several quotients are the decimals of their respective shillings.

II. The decimals of 11, 10, 9, &c. ſhillings, are divided by 12, and the quotients are the decimals of the pence 11, 10, 9, &c.

III. The decimals of 3, 2, 1, pence, are divided by 4, and the quotients are the decimals of the farthings 3, 2, 1.

39. Construction of Table II.

I. The ounces 11, 10, 9, 8, &c. are divided by 12, the ounces in a troy pound, and the quotients are the decimals of thoſe ounces.

II. The decimal of one ounce, is divided by 20, (the penny weights in one ounce) and the quote is the decimal of 1 penny weight; which multiplied by the other penny weights, gives their reſpective decimals. And from theſe are the decimals of grains, conſtructed in the ſame manner.

40. Construction of Table III.

I. Becauſe 20 hundred is one ton, therefore the decimals of the ſhillings will ſerve for the hundred weights, ſuppoſing 1 ton the integer.

II. The decimal of one ſhilling (*i. e.* 1 hundred weight) is divided by 4 (the quarters in one hundred weight) and the quote, is the decimal of 1 quarter, from whence the decimals of the other quarters are obtained, as are alſo the decimals of pounds.

41. Hundred Weight the Integer.

The decimals of the quarters are ,25, ,5 and ,75; and dividing the decimal of 1 quarter by 28, (the Pounds in a quarter) gives the decimal of 1 pound;

pound; from whence the decimals of the other pounds are obtained; as are alſo the decimals of ounces; and from them the decimals of drams.

42. One Pound *the* Integer.

One ounce is divided by 16, (the ounces in 1 pound) and it gives the decimal of 1 ounce; from whence the decimals of the other ounces are obtained; as are alſo the decimals of drams.

After the ſame manner are the other Tables of meaſures and time conſtructed; having always a due regard how many of a leſſer denomination are contained in a ſuperior one.

As theſe Tables are only carried to ſix places (excepting in ſome particulars) whenever the ſeventh figure would have been more than a 5, the ſixth place has been increaſed with unity; otherwiſe, the ſixth place is given as it ariſes in the work.

The uſe of theſe Tables are obvious; but to prevent all doubts obſerve the following

43. *EXAMPLES.*

I. *What is the decimal value of* 7 *oz.* 16 *dwt.* 18 *grs.*?

In Table II.

againſt	7 *oz.*	is	,5833333
againſt	16 *dwt.*	is	,0066666
againſt	18 *grs.*	is	,0031250

Theſe added, their ſum is 0,653125, the decimal required.

 II. *What*

II. *What is the decimal value of 46 gallons and 5 pints?*

In TABLE IV.

against	40 gallons	is	,634920
against	6 gallons	is	,095238
against	5 pints	is	,009921

These added, their ſum is 0,749079, the decimal required.

After the ſame manner are decimals of other denominations collected.

N. B. If the number of parts wanted are not found in the Tables, take it out at twice; thus: For 17 *dwt.* add 10 and 7 together; and the like for any other.

TABLE

TABLE I.

Coin, 1 lb the Integer.

S.	Dec.	S.	Dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	Decimals.
11	,04583 .
10	,0416 . .
9	,0375 . .
8	,03 . . . .
7	,02916 .
6	,025 . . .
5	,02083 .
4	,016 . . .
3	,0125 . .
2	,0083 . .
1	,00416 .

Farthings.	Decimals.
3	,003125
2	,00208 3
1	,00104 6

Note. *The following Table of Ounces will serve for Inches and any thing where 12 is the Integer.*

TABLE II.

Troy Weight.

1 lb the Integer.

Ounces.	Decimals.
11	,916 . . .
10	,83 . . . .
9	,75 . . . .
8	,6 . . . . .
7	,583 . . .
6	,5 . . . . .
5	,416 . . .
4	,3 . . . . .
3	,25 . . . .
2	,16 . . . .
1	,083 . . .

Dwts.	Decimals.
10	,0416 . .
9	,0375 . .
8	,03 . . . .
7	,02916 .
6	,025 . . .
5	,02083 .
4	,016 . . .
3	,0125 . .
2	,0083 . .
1	,00416 .

Gs.	
10	,001736
9	,001563
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

TABLE III.

Avoird. 1 *Ton the Integ.*

Qrs. Cwts.	*Decimals.*
3	,0375 . .
2	,025 . . .
1	,0125 . .
Pounds.	*Decimals.*
20	,008928
10	,004464
9	,004018
8	,003571
7	,003125
6	,002678
5	,002232
4	,001785
3	,001339
2	,000892
1	,000446

1 *Pound the Integer.*

Ounces.	*Decimals.*
10	,625 . . .
9	,5625 . .
8	,5 . . . . .
7	,4375 . .
6	,375 . . .
5	,3125 . .
4	,25 . . . .
3	,1875 . .
2	,125 . . .
1	,0625 . .

TABLE III.

Avoird. 1 ℔ *the Integ.*

Drams.	*Decimals.*
10	,039062
9	,035156
8	,031250
7	,027344
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

1 *Cwt. the Integer.*

Pounds.	*Decimals.*
20	,178571
10	,089285
9	,080357
8	,071428
7	,0625
6	,053571
5	,044642
4	,035714
3	,026785
2	,017857
1	,008928
Ounces.	*Decimals.*
10	,005580
9	,005022
8	,004464
7	,003906
6	,003348
5	,002790
4	,002232
3	,001674
2	,001116
1	,000558

TABLE III.

Avoirdupoise.

1 Cwt the Integer.

Drams.	*Decimals.*
10	,000349
9	,000314
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034

TABLE IV.

Liquid Measure.

1 Tun the Integer.

Gallons.	*Decimals.*
200	,793651
100	,396825
90	,357141
80	,317460
70	,27....
60	,238095
50	,198412
40	,158730
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027...
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

TABLE V.

Liquid Measure.

1 Ton the Integer.

Pints.	*Decimals.*
7	,003472
6	,002976
5	,002480
4	,001984
3	,001488
2	,000992
1	,000496

1 Hogshead the Integ.

Gallons.	*Decimals.*
60	,952381
50	,793651
40	,634921
30	,476190
20	,317460
10	,158730
9	,142857
8	,126984
7	,1.....
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints.	*Decimals.*
7	,013889
6	,011905
5	,009921
4	,007937
3	,005952
2	,003968
1	,001984

TABLE V.

Long Measure.

1 Mile the Integer.

Yards.	Decimals.
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

TABLE V.

Long Measure.

1 Mile the Integer.

Feet.	Decimals.
2	,0003787
1	,0001894

Inches.	Decimals.
9	,0001421
6	,0000947
3	,0000474
1	,0000158

Yard the Integer.

2	,6 . . . . .
1	,3 . . . . .

Inches.	Decimals.
11	,305 . . .
10	,27 . . . .
9	,249 . . .
8	,2 . . . . .
7	,194 . . .
6	,16 . . . .
5	,138 . . .
4	,1 . . . . .
3	,083 . . .
2	,05 . . . .
1	,027 . . .

Qrs. Inch.	Decimals.
3	,02083 .
2	,0138 . .
1	,00694 . .

TABLE VI.
Time.
1 Year the Integer.

Days.	*Decimals.*
300	,821918
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013698
4	,010959
3	,008219
2	,005479
1	,002740

TABLE VI.
Time.
1 Year the Integer.

Hours.	*Decimals.*
20	,002282
10	,001141
9	,001027
8	,000913
7	,000799
6	,000685
5	,000571
4	,000456
3	,000342
2	,000228
1	,000114
Minutes.	*Decimals.*
50	,0000951
40	,0000761
30	,00005706
20	,00003804
10	,00001902
9	,00001711
8	,00001521
7	,00001331
6	,00001141
5	,00000951
4	,00000761
3	,00000571
2	,00000380
1	,00000190
Qrs. Min.	*Decimals.*
3	,00000142
2	,00000095
1	,00000047

TABLE VI.

Time.

1 *Day the Integer.*

Hours.	*Decimals.*
20	,83
10	,416
9	,375
8	,3
7	,2916
6	,25
5	,2083
4	,16
3	,125
2	,083
1	,0416

Minutes.	*Decimals.*
50	,03472
40	,027
30	,02083
20	,0138
10	,00694
9	,00625
8	,005
7	,004861
6	,00416
5	,003472
4	,0027
3	,002083
2	,00138
1	,000694

Seconds.	*Decimals.*
45	,0005208
30	,0003472
15	,0001736

TABLE VII.

Various Measures.

1st. *Cloth Measures.*

1 *Yard the Integer.*

Qrs. Yard.	*Decimals.*
3	,75
2	,5
1	,25

Nails.	*Decimals.*
3	,1875
2	,125
1	,0625

Measure.

Liquid. | *Dry.*

Integer.

1 *Gall.* | 1 *Quarter.*

Pints.	*Decim.*	*Bushel.*
7	,875	7
6	,75	6
5	,625	5
4	,5	4
3	,375	3
2	,25	2
1	,125	1

Qs. Ps.	*Decim.*	*Pecks.*
3	,09375	3
2	,0625	2
1	,03125	1

Decimals.	*Qrs. Pecks.*
,023437	3
,015625	2
,007812	1

Decimals.	*Pints.*
,005859	3
,003906	2
,001953	1

SECTION VIII.

44. *Of* Comparison *or* Proportion.

THE comparing of things, of a like kind to one another, may be considered two ways:

First, By how much one thing exceeds, or is greater than another, and this is called *difference*.

Secondly, What part or parts one thing is of another, and this is called *ratio*. And two ratios make a *proportion*, viz.

When one number is to be divided by another, then the ratio of the divisor to the dividend, is the same as the ratio of unity to the quotient.

Or, the divisor is said to be in the same proportion to the dividend; as unity, is in proportion to the quotient; that is, the divisor is as often contain'd in the dividend, as unity is contain'd in the quotient.

Therefore the ratio of one number to another, is nothing more, than how often that number does contain, or is contain'd in, the other; or it is measured by the quotient arising from the division of one number by the other.

45. There cannot be less than two numbers or terms in any ratio; the first of which, or the term by which the comparison is made, is called the *antecedent*; and the second, or the term to which the first is compared, is called the *consequent*.

46. When two ratios (*i. e.* quotients) are equal, the numbers, or terms, to which these ratios belong, are said to be geometrically proportional.

Thus,

Thus, the ratio of 3 to 12, being the ſame as the ratio of 4 to 16; therefore the numbers 3, 12; 4, 16, are ſaid to be proportionals.

47. When of ſeveral terms, or numbers, the quotient of the firſt and ſecond is the ſame with that of the 2d and 3d, and the ſame with that of the third and fourth, *&c.* thoſe numbers are ſaid to be in continued geometric proportion: Thus, 4, 12, 36, 108, *&c.* are numbers in continued geometric proportion, for the quotient of any two adjacent terms is 3.

48. If there are three terms in geometrical proportion, the firſt term multiplied by the third, gives a product equal to the ſecond term multiplied by itſelf.

Hence the ſecond term is called *a mean proportional between the extreams*: viz. *between the firſt and third terms.*

49. If there are four terms in geometrical proportion, the product of the two extream terms is equal to the product of the two mean terms.

Hence the ſecond and third terms are called *mean proportionals between the firſt and fourth terms.*

Now it is evident, that if the product of the ſecond and third terms be divided by the firſt term, the quotient will be the fourth term.

And hence ariſes the method of operating the rule of three.

50. The rule of three, is ſo called, becauſe three numbers or terms are given to find a fourth proportional: thus, if the numbers 3, 12, and 18, were given to find a fourth proportional.

Multiply the ſecond term by the third, or (which is all one) the third by the ſecond, *viz.* 18 by 12, the Product will be 216; this 216 divided by the

firſt

first term 3, gives 72 in the quotient, for the fourth term.

Now by comparing of the terms together, it will be found, that the second term 12, as often contains the first term 3; as the fourth term 72, contains the third term 18: also that the third term 18, as often contains the first term 3, as the fourth term 72, contains the second term 12; that is, the ratio or proportion of 3 to 12, is the same as the ratio of 18 to 72. Also the ratio of 3 to 18, is the same as the ratio of 12 to 72; and the like in other numbers.

51. The rule of three being mostly concerned in finding a fourth number proportional to three numbers or terms given, differing in signification and denomination, the greatest difficulty lies in stating the terms; that is, in placing them in proper order to be multiplied and divided according to the foregoing directions; to do this proceed as follows.

52. Place that for the second, which has the same name with the fourth, or term sought.

Then consider, from the nature of the question, whether that fourth term should be less, or greater than the second.

If the second should be $\left\{ \begin{matrix} \text{less} \\ \text{greater} \end{matrix} \right\}$ than the 4th,

let the $\left\{ \begin{matrix} \text{least} \\ \text{greatest} \end{matrix} \right\}$ of the remaining two terms be placed first; and the other, for the third term; then the three given terms are truly stated.

If any, or all, of the terms be of different denominations, reduce the lesser ones to the decimal part or parts of the greater, as shewn in Sect. II. observing, that the first and third terms, be reduced to the decimal part or parts of one and the same denomination.

Multiply

Multiply the second and third terms together, by the rules in Sect. IV. divide the product by the first term, adjust the quotient by the rules given in Sect V. and this will be the fourth term, or the number sought, and is to be valued by the rules given in Sect. II.

QUESTION I.

What will 326 ¼ ℔ of tobacco come to, at 3 *s.* 6 *d.* for 1 ½ ℔?

Here the quality of the fourth term is money, *viz.* the worth of 326 ¼ ℔ of tobacco; and among the three terms given, one is money, *viz.* 3 *s.* 6 *d.* the worth of 1 ½ ℔.

Now it is easy to see, that 326 ¼ ℔ will come to more than 1 ½ ℔; therefore the fourth term will be greater than the second; and the terms stated, will stand thus: if 1 ½ ℔ cost 3 *s.* 6 *d.* what will 326 ¼ ℔ come to; and the terms reduced into decimals will stand thus:

```
If 1,5 ℔—0,175 l.—326,25 ℔
                    0,175
                  -------
                  163125
                 554625
                 -------
     1,5)  57,09375 (38,0625 l.
           120
            93
             37
              75
```

Here the third term being multiplied by the second, and the product being divided by the first term,

term, gives 38,0625 *l.* in the quotient; which being valued as directed at Art. 19, gives 38 *l.* 1 *s.* 3 *d.* the money that 326 ¼ ℔ of tobacco will come to, at the rate of 3 *s.* 6 *d.* for 1 ½ ℔.

QUESTION II.

What is the worth of 19 *oz.* 3 *dwts.* 5 *grs.* of gold, at 2 *l.* 19 *s.* 4 *d.* *per ounce*?

If 1 *oz.*—2 *l.* 19 *s.* 4 *d.*—19 *oz.* 3 *dwts.* 5 *grs.*

$$\begin{array}{r|l} & 5 \\ 24\left\{\begin{array}{l}4\\6\end{array}\right. & (125 \\ 20 & 3,208\cancel{3} \\ & 0,16041\cancel{6} \end{array} \qquad \begin{array}{r|l} 12 & 4 \\ 20 & 19,\cancel{3} \\ & 2,9\cancel{6} \end{array}$$

Or thus in Decimals.

If 1 *oz.*——2,9$\cancel{6}$ *l.*——19,16041$\cancel{6}$ *oz.*

$$\begin{array}{rr} & 19,16041\cancel{6} \\ & 2,9\cancel{6} \\ \hline 9) & 1149\,6\,2500 \\ \hline & 127736\cancel{1}11 \\ & 17244375\cancel{0}0 \\ & 38320833\cancel{3}33 \\ \hline & 56,842569\cancel{4} \end{array}$$

Because the first term here is unity or 1, which neither multiplies nor divides, therefore the answer is produced by multiplying the second and third terms together; and the product being valued, as shewn in Art. 19, gives 56 *l.* 16 *s.* 10 ¼ *d.* the value of the gold, as sought after.

QUES-

QUESTION III.

What is the worth of 827 $\frac{3}{4}$ *yards* of painting at 10$\frac{1}{2}$ *d. per yard.*

If 1 *yd.*—0,04375 *l.*—827,7500 *yd.*

$$\begin{array}{r} 57\,340{,}0 \\ \hline 33\,1100 \\ 2\,4832 \\ 5794 \\ 414 \\ \hline 36{,}2140 \end{array} \left\{ \begin{array}{l} \text{Answ. } 36\,l. \\ 4\,s.\ 3\tfrac{1}{2}\,d. \end{array} \right.$$

Now because there would be 7 decimal places in the answer, whereof 4 are more than sufficient, therefore to get 4 decimals in the product, the place of o units is put under the fourth decimal in the multiplicand, and the order of the rest inverted, as directed at Art. 28.

QUESTION IV.

Lent my friend 34 *l.* for $\frac{5}{8}$ of a year, how much ought he to lend me $\frac{9}{12}$ of a year, to requite my kindness?

0,75 *y.*—34 *l.*—0,625 *y.*

$$\begin{array}{r} 34 \\ \hline 2500 \\ 1875 \\ \hline 75)\ 21{,}25\ (28{,}333 \\ 625 \\ \hline 250 \\ \hline 25 \end{array}$$

Answer. 28 *l.* 6 *s.* 8 *d.*

QUES-

QUESTION V.

If $\frac{3}{4}$ of a yard of cloth, that is 2 $\frac{1}{4}$ yards broad, make a garment; how much of another sort, that is but $\frac{4}{5}$ of a yard wide, will make the same garment?

```
                          ,75
0,8 y. b.—0,75 y. l.—2,25 y. b.
                      ————
                         1125
                        1575
                        ————
              ,8) 1,6875 (2,1093, &c.
                  ————
```

Answer, 2 yards and 1 nail.

QUESTION VI.

If when the bushel of wheat cost 4 *s.* 9 *d.* the penny loaf weigh'd 10 $\frac{2}{3}$ ounces; what should it weigh, when the bushel of wheat is sold for 8 *s.* 10 *d.*

```
 £.      oz.      £.
0,4416—10,6—0,2375
                10,6
            ————————
           9)14250
            ————————
              15833
             23750
            ————————
      0,4416)2,53
         441) ,25
            ————————
      0,3975)2,2800(5,72, &c. Ounces.
            ————————
              2925
             ————
               143
```

The foregoing method includes both the direct and inverse rules of three: But the following

rule, comprehends all cases that come under the rules of three, whether direct or inverse, whether single or double, or any how compounded; and is of such general use, as to extend to all arithmetical operations, where proportions are concerned: It was reduced to this form about the year 1706, by the celebrated *William Jones*, Esq; F. R. S. and has ever since been commonly known, and made use of by mathematicians.

52. The Rule of Proportion.

1. *Set down the terms expressing the condition of the question, in one line.*
2. *Under each conditional term, set its corresponding one, in another line.*
3. *Multiply the producing terms of one line, and the produced term of the other line, continually; and take the result for a dividend.*
4. *Multiply the remaining terms continually, and let the product be a divisor.*
5. *The quotient of this division, will be the term required.*

By producing terms, here, is meant, whatever necessarily and jointly produce any effect; as, the cause and the time; length, breadth and depth; buyer and his money; seller and his goods; things carried and their distance; exchanger and the things exchanged, &c. all necessarily inseparable in producing their several effects.

In a question where a term is only understood, and not expressed, that term may ever be expressed by unity.

A term expressed by a number having different names, must be reduced so as to have the same name.

A quo-

A quotient is represented by the dividend put above a line, and the divisor put below it.

EXAMPLES.

I. If 150 £. serve 4 persons for 16 weeks: What sum will serve 13 persons for 49 weeks?

£.	P.	W.
150 ---	4 ---	16
Q. ---	13 ---	49

Now $Q. = \left(\frac{150 \times 13 \times 49}{4 \times 16} =\right)$ £. 1492 s. 19 d. $4\frac{1}{2}$

For

$$
\begin{array}{rrl}
 & 150 & \\
 & 13 & \\
\hline
 & 450 & \\
 & 150 & \\
\hline
 & 1950 & \\
 & 49 & \\
\hline
16 & 17550 & \\
4 & 7800 & £. \\
\hline
64) & 95550 & (1492,96875 \\
 & 315 & \\
 & 595 & \\
 & 190 & \\
 & 620 & \\
 & 440 & \\
 & 560 & \\
 & 480 & \\
 & 320 & \\
\end{array}
$$

II. If

II. If 40 acres of graſs be mowed by eight men in ſeven days: How many acres can be mowed by twenty-four men in twenty-eight days?

A.	M.	D.
40 ---	8 ---	7
Q. ---	24 ---	28

Now Q. $= \left(\frac{40 \times 24 \times 28}{8 \times 7} = \right)$ 480 acres.

III. If the carriage of 126 lb for 200 miles, coſt 6 ſhillings: How many lb will be carried 750 miles for 20 ſhillings?

lb.	M.	S.
126 ---	200 ---	6
Q. ---	750 ---	20

Now Q. $= \left(\frac{126 \times 200 \times 20}{750 \times 6} = \right)$ 112 lb.

IV. If 14 horſes in 16 days eat 56 buſhels of oats: How many buſhels will 20 horſes eat in 24 days?

H.	D.	B.
14 ---	16 ---	56
20 ---	24 ---	Q.

Now Q. $= \left(\frac{20 \times 24 \times 56}{14 \times 16} = \right)$ 120 buſhels.

V. How

V. How many yards of bays of 3 quarters wide, will suffice to line 1000 Soldiers coats, each containing 2½ yards of cloth, of 5 quarters wide?

yds. lo.	qrs wi.	coat
2,5 ---	5 ---	1
Q. ---	3 ---	1000

Now $Q. = \left(\frac{2,5 \times 5 \times 1000}{3} = \right) 4166,\not{6}$ yds.

VI. What is the interest of 542 £. 10 s. for 219 days, at the rate of 5 £. *per cent. per annum.*

£. prin.	*Days*	*£. int.*
100 ---	365 ---	5
542,5 ---	219 ---	Q.

Now $Q. = \left(\frac{542,5 \times 219 \times 5}{100 \times 365} = \right)$ 16 £. 5 s. 6 d.

VII. A footman can run 240 miles in 4 days, of 12 hours long: How many days of 16 hours long, will he be in running 720 miles?

Footm.	Da.	Ho.	miles
1 ---	4 ---	12 ---	240
1 ---	Q. ---	16 ---	720

Now $Q. = \left(\frac{4 \times 16 \times 720}{12 \times 240} = \right)$ 9 days.

VIII. If 12 measures of wine, at 20 *d.* each, serve 8 men for 3 days: How many measures at 16 *d.* each, will serve 24 men for 2 days?

Mea.	Pence	M.	Da.
12	20	8	3
Q.	16	24	2

$$\text{Now } Q. = \left(\frac{12 \times 20 \times 24 \times 2}{16 \times 8 \times 3} = \right) 30 \text{ meas.}$$

IX. If a garrison of 3600 men have bread for 35 days, at 24 oz, each a day: How much a day may be allowed to 4800 men, each for 45 days, that the same quantity of bread may serve?

Men	oz.	da.	Bread.
3600	24	35	1
4800	Q.	45	1

$$\text{Now } Q. = \left(\frac{3600 \times 24 \times 35}{4800 \times 45} = \right) 14 \text{ oz. } per \text{ day.}$$

X. If when the tun of wine is worth 30 £; 20 £ worth will serve a ship's company of 336 men for 4 days, at a pint to each a day: How long will 500 £ worth serve a crew of 250 men, at 1 $\frac{1}{2}$ pint to each man a day; when the tun is worth 24 £.

Tun	£.	M.	da.	Pint	£. w.
1	30	336	4	1	20
1	24	250	Q.	1,5	500

$$\text{Now } Q. = \left(\frac{30 \times 336 \times 4 \times 500}{24 \times 250 \times 1{,}5 \times 20} = \right) 112 \text{ days.}$$

XI. If

XI. If when the quarter of wheat is sold for 2 £. 12 *s.* 6 *d.* the three penny loaf weighs 22 *oz.* 16 *dwts.* What ought the 2 penny loaf to weigh, when the bushel is sold for 5 shillings?

Now 2 £. 12 *s.* 6 *d.* = 2,625 £; and 22 *oz.* 16 *dwts.* = 22,8 *oz.*

Quar.	£.	pen. lo.	oz.
1	2,625	3	22,8
1	2,	2	Q.

$$\text{Then Q.} = \left(\frac{2{,}625 \times 22{,}8 \times 2}{2 \times 3} = \right) 19{,}95 \text{ oz.}$$

XII. If 48 men in 5 ½ days, dig a trench of 23 ¼ yards long 2 ⅓ deep, and 3 ⅔ wide: What length of trench, of 3 ½ yards deep, and 5 ⅗ wide, can be dug by 24 men in 189 days?

Men	da.	yds. lo.	deep	wide
248	5,5	23,25	2,3	3,6
24	189	Q.	3,5	5,6

$$\text{Now Q.} = \left(\frac{24 \times 189 \times 23{,}25 \times 2{,}3 \times 3{,}6}{248 \times 5{,}5 \times 3{,}5 \times 5{,}6} = \right) 33{,}75 \text{ yds. lo.}$$

XIII. If 14 yards of cloth cost 10 guineas : How many ells flemish may be had for 383 £. 17 s. 6 d.?

Man £. Ell Fl. *
1 - - 10,5 - - 14 × 1,3
1 - - 383,875 - - - Q.

$$\text{Then } Q. = \left(\frac{14 \times 1{,}3 \times 383{,}875}{10{,}5} = \right) \begin{matrix} £. & s. & d. \\ 504 & 13 & 4 \end{matrix}$$

XIV. If 13 ells of diaper of $\frac{3}{4}$ yard wide, cost 5 guineas : What will 32 $\frac{1}{4}$ yards of $\frac{3}{4}$ ell english wide, and of the same goodness, come to?

Man £. † yds. lo. wide
1 - - - 5,25 - - - 13 × 1,25 - - - 0,75
1 - - - Q. - - - - 32,25 - - - 0,6 × 1,25

$$Q. = \left(\frac{32{,}25 \times 0{,}6 \times 1{,}25 \times 5{,}25}{13 \times 1{,}25 \times 0{,}75} = \right) \begin{matrix} £. & s. \\ 8 & 12 \end{matrix}$$

---

* *Note*, Yds. multipl. by $\left(\frac{4}{3} = \right)$ 1,3 gives ells flem.

† *Note*, ells engl. multip. by $\left(\frac{5}{4} = \right)$ 1,25 gives yds.

XV. One

XV. One who had ſold a parcel of cloths at 2 *s.* 10 *d.* a yd. on 3 mon. credit, found he had gain'd 25 £. *per cent. per. annum*: What did the cloth coſt a yard?

$$\begin{array}{ccc} \pounds. & \text{mon.} & \pounds. \\ 100 \text{ - - - } & 12 \text{ - - - } & 25 \\ 100 \text{ - - - } & 3 \text{ - - - } & Q. \end{array}$$

Now $Q. = \left(\frac{100 \times 3 \times 25}{100 \times 12} = \right) 6{,}25\ \pounds.$

And $\overset{\pounds.}{100} + \overset{\pounds.}{6{,}25} = \overset{\pounds.}{106{,}25}$. Alſo, $\overset{s.\ d.}{2\ 10} = 0{,}1416\ \pounds.$

$$\begin{array}{lccc} & \pounds. & \text{Mon.} & \pounds. \\ \text{Then} & 106{,}25 \text{ - - - } & 3 \text{ - - - } & 100 \\ & 0{,}1416 \text{ - - } & 3 \text{ - - - } & Q. \end{array}$$

Therefore $Q. = \left(\frac{0{,}1416 \times 3 \times 100}{106{,}25 \times 3} = \right) \overset{\pounds.}{0{,}13}$

Or, 2 *s.* 8 *d.* a yard is the prime coſt.

XVI. At 3 *s.* 4 *d.* a pound: What will 754 $\frac{1}{2}$ lb be worth, allowing 4 lb upon every 100 pound?

Man	lb	lb
1	104	100
1	754,5	Q.

$$\text{Now Q.} = \left(\frac{754,5 \times 100}{104} = \right) 725,4807 \text{ lb.}$$

Man	lb	£.
1	1	$0,1\dot{6}$
1	725,4807	Q.

$$\text{Then Q.} = (725,4807 \times 0,1\dot{6} =)\ 120\,£.\ 18\,s.\ 3\tfrac{1}{2}\,d.$$

XVII. One would exchange 729 pieces of 4 *s.* 2 *d.* each, for pounds sterling, and must allow the broker 1$\frac{1}{4}$ £. upon 100 £. How many £. will he receive?

Man	Piece	£.
1	1	$0,208\dot{3}$
1	729	Q. $= 729 \times 0,208\dot{3}$ £.

Man	£.	£.
1	101,25	100
1	$729 \times 0,208\dot{3}$	Q.

$$\text{Q.} = \left(\frac{729 \times 0,208\dot{3} \times 100}{101,25} = \right) 150\,£.$$

XVIII. A

XVIII. A draper bought 27 pieces of cloth, each of 24 ½ yards long, and 7 quarters wide, at 14 *s.* 8 *d.* a yard: How many pieces of such cloth, each of 31 ¼ yards long, and 5 quarters wide, may be bought for 1375 £.?

yd. lo.	q. w.	£.
1	7	$0,7\dot{3}$
27 × 24,5	7	Q. $=27\times 24,5\times 0,7\dot{3}$ £.

Pieces	yds. lo.	q. w.	£.
27	24,5	7	$27\times 24,5\times 0,7\dot{3}$
Q.	31,25	5	1375.

Now $Q. = \left(\frac{7\times 1375}{31,25\times 5\times 0,7\dot{3}}=\right)$ 84 Pieces.

XIX. A grocer bought 4 ¼ hundred weight of pepper, for 15 £. 17 *s.* 4 *d.* which proving to be damaged, he is willing to lose 12 ½ £. *per cent.* How must he sell it a pound?

Man	£.	£.
1	100	87,5
1	$15,8\dot{6}$	Q.

$$Q. = \left(\frac{15,8\dot{6}\times 87,5}{100}=\right) \; 15,8\dot{6}\times 0,875 \text{ £.}$$

Man	lb	£.
1	476	$15,8\dot{6}\times 0,875$
1	1	Q. $=\frac{15,8\dot{6}\times 0,875}{476}$ £.

He must sell it at 7*d.* a lb.

XX. A merchant sent to his factor at *Lisbon* 36 cloths, each of 22 ½ yards, at 9 *s.* 6 *d.* a yard; who is to return the third part in sugar at 22 *s.* 6 *d.* a hundred weight, and the rest in wine, at 13 £. 10 *s.* a pipe: What quantity of sugar and wine did the factor send?

Man yd. £.
1 - - - - - - 1 - - 0,475
1 - - - 22,5 × 36 - Q.

$$Q. = (36 \times 22,5 + 0,475 =)\ 384,75\ £.$$

Then 384,75 £. is the value of the cloth.
And 128,25 £. is the sum returned in sugar.
Also 256,5 £. is the sum returned in wine.

Man £. hun. wt.
1 - - 1,125 - - 1
1 - - 128,26 - - $Q. = \left(\frac{128,25}{1,125} =\right)$ 114 hundr. wt. of sugar

Man £. Pipe
1 - - 13,5 - - 1
1 - 256,5 - - $Q. = \left(\frac{256,5}{13,5} =\right)$ 19 pipes of wine.

53. SECTION IX.

FOR the ſpeedy working of thoſe examples in the rule of three, where the firſt term is an unit; there are, in books of arithmetic, ſeveral compendious Rules, called the *Rules of Practice*, which are nothing more than the application of the doctrine of *aliquot parts:* But as there are a great variety of ſuch parts; ſo many, therefore, are the ways of applying them; which occaſions ſuch a diverſity of Rules for doing one and the ſame thing, that it would be an endleſs taſk, to give all the eaſy methods of operation adapted to particular Caſes; in this place it may ſuffice to mention ſome general directions, whereby, with the judgment of the practitioner, the moſt common Caſes may expeditiouſly be ſolved.

One number is ſaid to be an *aliquot part* of another; when the former will divide the latter, and leave no remainder.

Tables, ſhewing the aliquot parts of a pound ſterling, and of a ſhilling.

10*s*. 0*d*. is	$\frac{1}{2}$	Of a pound ſterling.	6*d*. is	$\frac{1}{2}$	Of a ſhilling.	Or, $\frac{1}{40}$	Of a pound.
6*s*. 8*d*.	$\frac{1}{3}$		4*d*.	$\frac{1}{3}$		$\frac{1}{60}$	
5*s*. 0*d*.	$\frac{1}{4}$		3*d*.	$\frac{1}{4}$		$\frac{1}{80}$	
4*s*. 0*d*.	$\frac{1}{5}$		2*d*.	$\frac{1}{6}$		$\frac{1}{120}$	
3*s*. 4*d*.	$\frac{1}{6}$		$1\frac{1}{2}$*d*.	$\frac{1}{8}$			
2*s*. 6*d*.	$\frac{1}{8}$		$\frac{3}{4}$*d*.	$\frac{1}{16}$			
2*s*. 0*d*.	$\frac{1}{10}$						

The

The solution of all the Cases in Practice, may be performed by the following,

54. General RULE.

Of the greater denominations of the things given to be valued.

1. Take the product thereof by the pounds in the price.
2. And also, the parts thereof, for the most convenient aliquots of the lower denominations of the price.
3. And the parts of the given price, for the most convenient aliquots of the lower denominations of the quantity.
4. The sum of these, is the value of the quantity given.

In the following Examples, the aliquot parts, or divisors, are set against the numbers they are to divide.

Ex. I. At 3*s.* 6*d.* a pair, what will 273 pair come to?

s.	*d.*					
2	0 is	$\frac{1}{10}$	273			
			27	6		
1	0	$\frac{1}{2}$	13	13		
0	6	$\frac{1}{2}$	6	16	6	
		£.	47	15	6	

Ex.

Ex. II. At 17 *s.* 10 *d.* ½ a yard; what will 483 ¾ yards come to?

		s.	*d.*					
		10	0	is	½	483		
		5	0		½	241	10	
		2	6		½	120	15	
		0	3		$\frac{1}{10}$	60	7	6
Y.		0	1½		½	6	0	9
½	½	17	10½			3	0	4½
¼	½	8	11¼	- - -			8	11¼
		4	5⅝	- - -			4	5⅝
						432	7	0¼

Ex. III. At 48 £. 14 *s.* 7 *d.* ¼ *per C. wt.*; what will 596 *C. wt.* 3 *qrs.* 19 ℔ ½ come to?

		£.	*s.*	*d.*						
		0	10	0	is ½		596			
							48			
							4768			
							2384			
		48	0	0			28608			
		0	2	0	$\frac{1}{5}$		298	the ½ of 596		
		0	2	0	$\frac{1}{1}$		- - 59	12		
		0	0	6	$\frac{1}{4}$		- - 59	12		
		0	0	1½	$\frac{1}{4}$		- - 14	18		
qrs.	℔	0	0	0¼	$\frac{1}{6}$		- - - 3	14	6	
2	0 is ½	48	14	7¼			- - - 0	12	5	
1	0	$\frac{1}{1}$	- - - - -				24	7	3¾,5	
0	14	$\frac{1}{2}$	- - - - -				12	3	7¾,75	
0	2	$\frac{1}{7}$	- - - - -				6	1	9¾,87	
0	2	$\frac{1}{1}$	- - - - -				0	17	4¾,41	
0	1	$\frac{1}{2}$	- - - - -				0	17	4¾,41	
0	0½	$\frac{1}{2}$	- - - - -				0	8	8¼,7	
							0	4	4 ,85	
							£. 29089	9	7 ,49	

But

But most Examples of this kind, are more readily solved, by mixing the doctrine of decimal fractions, with that of aliquot parts; as shewn by the following,

55. General RULE.

1. To the greater denomination of the things given to be valued, annex the decimal of the inferior parts (if any.)
2. Take the product of this mix'd number, by the pounds in the price.
3. And also, the product of $\frac{1}{10}$ thereof, by half the greatest even shillings in the price.
4. Out of the said $\frac{1}{10}$ of the things given to be valued, take the most convenient aliquots of 2 shillings, for the other parts (if any) of the price.
5. The sum of the lines produced by the 2d, 3d, and 4th articles, will be the answer decimally expressed.

Ex. I. At 3 *s.* 6 *d.* a pair, what will 273 pair come to?

27,3 is $\frac{1}{10}$ of 373.

s.	*d.*			1 is $\frac{1}{2}$ the greatest even shillings.
1	0	$\frac{1}{2}$	27,3	the value at 2 shillings.
0	6	$\frac{1}{2}$	13,65	the value at 1 shilling.
			6,826	value at 6 pence.

£. 47,775 = 47 £. 15 *s.* 6 *d.*

Ex. II. At 17 *s.* 10 *d.* ½ a yard; what will 483 ¾ yards come to?

Now 483 ¾ is 483,75

s.	d.			
1	0	½	48,375	is $\frac{1}{10}$ of 483,75
			8	is ½ even shillings
			387,000	is the value at 16 shillings
0	6	½	24,1875	is the value at 1 shilling
0	3	½	12,09375	is the value at 6 pence
0	1½	½	6,04687	is the value at 3 pence
			3,02343	is the value at 1 ½ *d.*
			£. 432,35155	

Ex. III. At 48 £. 14 *s.* 7 ¾ *per C. wt.*: What will 596 *C. wt.* 3 *qrs.* 19 ½ lb come to?

2		1
28	4	19,5
	7	4,875
4		3,69643

s.	d.			
0	6	¼	596,92411	its $\frac{1}{10}$ is 59,692411
			48	
			477539288	
			238769644	
			28652,35728	
			417,846877	is 59,692411 by 7
0	1½	¼	14,923103	is ¼ of 59,69, &c.
0	0¼	$\frac{1}{6}$	3,730776	
			0,621796	
			29089,479832	

This Rule saves the trouble of taking the parts of the price, for the parts of the given things.

SEC-

SECTION X.

56. *Of Powers and their Roots.*

WHEN a given number is multiplied by itself, and this product by the given number, and this product by the given number, *&c.* to any assigned number of products; this process is called the involution of the given number, or the raising it to its powers.

Thus, the given number, is called the root or first power.

The 1st pow. multi. by itself, gives the 2d pow.
The 2d pow. multi. by the 1st, gives the 3d pow.
The 3d pow. multi. by the 1st, gives the 4th pow.
&c.

The 2d power is called the square.
The 3d power is called the cube.
The 4th power is called the biquadrat.
&c.

The following table exhibits the 1st, 2d, and 3d powers of the nine digits.

Roots.	1.	2.	3.	4.	5.	6.	7.	8.	9.
Squares.	1.	4.	9.	16.	25.	36.	49.	64.	81.
Cubes.	1.	8.	27.	64.	125.	216.	343.	512.	729.

From what has been said, it will be easy to find any assigned power of a given number; but when a large number is considered as a given power, and its root be required, this is not to be done so readily as

as the raising the power from the given root; in the latter Case, the factors are always known; but in the former, there is given, only the dividend, the divisor to which, is to be found; and this is not constant, as in Division, but changes every time a new figure or place is obtained in the quotient.

The method of finding the roots of given powers, is called, the Extraction of roots.

The common methods of performing the operations in the square and cube roots only, will here be shewn; those of higher powers not being much wanted in common mensuration: but the more inquisitive sort of readers may be amply satisfied in the methods of extracting the roots of higher powers, in almost every book of elementary Algebra.

57. *To extract the square root of any given number.*

RULE.

1. Put a point over the place of units; and also, over every second place (counting from units,) to the left-hand, for integers, and to the right-hand, for decimals; and the integral part of the root, will have as many places, as there are points over the integers in the given number.

When a number is thus pointed, the place under a point, and its left-hand place together, is called a period.

2. Seek the greatest square in the left-hand period, write the root in the quotient; the square thereof, write under the period; subtract, and to the remainder bring down the next period; (as in Division,) call this the resolvend.

3. To

3. To the left of the reſolvend, write the double of the quotient for a diviſor; ſeek how oft this may be had in the reſolvend, except its right-hand place; write the reſult in the quotient; and alſo on the right of the diviſor.

4. Multiply this increas'd diviſor, by the laſt quotient figure; ſubtract the product from the reſolvend; to the remainder bring down the next period for a new reſolvend; double all the quotient for a diviſor; divide as before; and thus proceed until all the periods are uſed.

If at laſt there happen to be a great remainder, and it is required to have the root more accurate, by increaſing it with a decimal fraction. To the remainder annex two cyphers, and proſecute the work as before, always adding two cyphers to the remainder, *&c.* till the root is as exact as deſired.

EXAMPLE I.

What is the ſquare root of 132496?

```
        ˙ ˙ ˙ ˙
        132496 (364
         9
   66)  424
        396
  724)  2896
        2896
```

The firſt period towards the left-hand is 13, the greateſt ſquare therein is 9, whoſe root 3, write in the quote, and the ſquare 9 write under the 13, and ſubtracting, there remains 4; to which bring down the next period 24, makes the reſolvend 424; to the left-hand thereof draw a curv'd line, and at ſome diſtance therefrom put the double of 3, *viz.*

viz. 6, and enquire how oft this 6 may be had in 42, and find 6 times, write 6 in the quote, and alſo, on the right hand of the diviſor 6, and the increaſed diviſor 66, multiply by the 6 in the quote, and it gives 396, this ſubtracted from 424 leaves 28; to which, the next period 96 is brought down, and 2896 is the new reſolvend; now 36 the quotient, doubled, makes 72 the diviſor, this in 289 goes 4 times, write 4 in the root, and diviſor; and the new diviſor 724, multiplied by 4, gives 2896, to be ſubtracted from the laſt reſolvend, and nothing remains: therefore 364 is the true root; for 364 multiplied by 364, gives 132496 the number given.

EXAMPLE II.

What is the ſquare root of 763958207163?

```
            763958207163) (874047,027
            64
     167)   1239
            1169
      1744)   7058
              6976
    174804)   822071
              699216
   1748087)    12285563
               12236609
  174809402)      489540000
                  349618804
 1748094047)      13992119600
                  12236658329
                   1755461281, &c.
```

In

In this Example, after all the periods in the given number are brought down and used; to the remainder are brought down periods of cyphers; and with these, the Work is prosecuted in the same manner as if they were given periods of significant digits: And thus, may the root be continued to almost any desired exactness; for there will ever be a remainder, since no digit squared, can have a cypher in its right-hand place.

More EXAMPLES.

What is the square root of

36372961	Answer.	6031
24681024		4968
1,0609		1,03
911236798,794365		30186,699

These Examples are operated in the same manner as the preceding ones.

There are many uses to which the square root may be applied, one is,

58. Two numbers being given, between them to find a mean proportional.

RULE.

Multiply the two numbers together, and out of the product extract the square root, which root is the mean proportional required.

EXAMPLE I.

What is the mean proportional between 3 and 12?

Now 3 multiplied by 12, is 36; whose square root is 6, the mean required.

For

For 3 is to 6, so is 6 to 12, by the rules of proportion.

EXAMPLE II.

Find a mean proportional between 4276 and 842?

```
   4276              3600392 (1897,4, &c.
    842              1
  -----            ----
   8552        28) 260
  17104            224
  34208          -----
  -------     369)  3603
  3600392            3321
                   ------
             3787)  28292
                    26509
                   -------
          3794,4)  178300
                   151776
                   -------
                    26524, &c.
```

So 1897,4, &c. is the mean proportional required.

59. *To extract the cube root of a given number.*

RULE.

1. Put a point over the place of units; and also, over every third place, counting to the left for integers, and to the right for decimals; or in other words, point the given number into periods of three places each, beginning at units: and there will be as many integral places in the root, as there are points over the integers in the given number.

2. Seek the greatest cube in the left-hand period, write the root in the quotient, and the cube under the

the period; subtract, and to the remainder bring down the next period: Call this the Resolvend, under which draw a line.

3. Under the Resolvend, write the triple square of the root, so that units in the latter stand under the place of hundreds in the former; under the triple square of the root, write the triple root, removed one place to the right; and the sum of these two lines call a Divisor; under which draw a line.

4. Seek how oft this Divisor may be had in the Resolvend, [its right-hand place excepted,] and write the result in the quotient.

5. Under the Divisor, write the product of the triple square of the root by the last quotient figure, setting units place of this line, under that of tens in the Divisor; under this line write, the product of the triple root, by the square of the last quotient figure, let this line be removed one place beyond the right of the former; and under this line, removed one place forward to the right, write the cube of the last quotient figure; the sum of these three lines call the Subtrahend, under which draw a line.

6. Subtract the Subtrahend from the Resolvend; to the remainder bring down the next period for a new Resolvend; the Divisor to this, must be the triple square of all the quotient added to the triple thereof, &c. as in the 3d Article, &c.

EX-

EXAMPLE I.

What is the Cube root of 48228544?

	48228544 (364		
	27		
	21228	Resolvend.	
add	27	Triple square of 3.	the root.
	09	Triple of 3. - -	
	279	Divisor.	
add	162	Triple square of 3 multiplied by 6.	
	324	Triple of 3 multipl. by square of 6.	
	216	Cube of 6.	
	19656	Subtrahend.	
	1572544	Resolvend.	
add	3888	Triple square of 36.	the root.
	108	Triple of 36. - -	
	38988	Divisor.	
add	15552	Triple square of 36 mult. by 4.	
	1728	Triple of 36 multi. by square of 4.	
	64	Cube of 4.	
	1572544	Subtrahend.	

If the work of this Example be well considered, and compared with the foregoing Rule, it will be easy to conceive how any other Example of the like nature may be wrought; and here observe, that when the cube root is extracted to more than two places, there is a necessity of doing some work upon a spare piece of paper, in order to come at the root's triple square, and the product of the triple root by the square of the quotient figure, *&c.*

In this example, the given number is a cube number, and therefore at the end of the operation

there remained nothing; for 364 multiplied by 364, the product multiplied by 364 gives 48228544, the given number.

But if the number given be not a cube number; then, to the last remainder always bring down three cyphers, and work anew for a decimal fraction if needful.

More EXAMPLES.

What is the cube root of

389017	Answers.	73
1092727		103
27054036008		3002
219365327791		6031
122615327232		4968

These examples are all operated in the same manner as the foregoing one.

60. There are many uses of the cube root, one is to find two mean proportionals between two given numbers.

RULE.

Divide the greater extream by the lesser, and the cube root of the quotient multiplied by the lesser extream, gives the lesser mean. Multiply the said cube root by the lesser mean, and the product is the greater mean proportional.

Note, This is only understood of those numbers that are in continued geometric proportion.

EXAMPLE I.

What are the two mean proportionals between 4 and 108?

108 divided by 4 gives 27, whose cube root is 3; and the lesser extream 4, multiplied thereby, gives 12 for the lesser mean; and 12 multiplied by the said root 3, gives 36 for the greater mean.

For as 4 is to 12, so is 36 to 108.

EXAMPLE II.

Find the two geometric means between 8 and 1728?

Now 8) 1728 (216, whose cube root is 6. And 6 times 8 is 48, the lesser mean, and 6 times 46 is 288 the greater mean.

For as 8 is to 48, so is 288 to 1728.

61. If the rule already given for the cube root be thought too tedious, the following one will be found more ready for use.

1. Point the given number, seek the greatest cube in the left-hand period, write the root in the quotient, subtract the cube from the period, as directed in the other rule; and to the remainder bring down all the remaining periods in the given number: Call this the resolvend.

2. To the root (or quotient) annex as many cyphers, as there are remaining periods; multiply this by 3; by this product, divide the Resolvend; and point the quotient into periods of 2 places, (beginning at units,) observing that there be no more points than there were periods brought down to the Resolvend.

3. Make the root (found in the first period of the given number,) a Divisor, seek how often it may be had in the left-hand period of the quotient [excepting the place under the point] and the

figure resulting write in the quotient, [to the right-hand of the root first found] and on the right of the divisor; multiply this increased divisor by the last quotient figure; to the remainder bring down the next period; divide this, by the last divisor, &c.

An example or two will render the whole very plain.

What is the cube root of 12812904?

```
        12812904  (200
        8            3
(6,00)  4812904    600
   23)  8021,5, &c.    (34
        69              200
  234)  1121            234
        936
        185
```

First begin at 4 the place of units, over which put a point, and omitting two figures, put another point over the 2; and omitting two figures more, put another point over the left-hand figure 2: Now here are three points, and therefore there must be three places of integers in the root. Then, beginning with the first period 12, find the greatest cube therein, which is 8, whose root is 2; write 8 under 12, and 2 as a quotient. Subtract 8 from 12, and to the remainder 4 bring down the remaining figures (which occupy the places of two points or periods.) To the quotient, (2) annex two cyphers, for the two points remaining over the given number, [for the quotient 2 is in reality 200,] and this 200, multiply by 3, and the product 600 make a divisor; by it divide 4812904, (which is the difference between 8 the cube of 2, and the given

given number,) and the quotient is 8021,5, &c. Then begin at 1, the place of units, and point as directed in the square, till there are as many points as annexed cyphers to the first root, which in this example are two. To the right and left of this number, *viz.* 8021, &c. draw curved lines, as in division; make the first root 2 a divisor, inquire how oft it may be found in the first period 80 [excepting the place under the point; that is, say how often 2 in 8,] and it gives 3, write this in the quotient; and also on the right-hand of the divisor 2, which now becomes 23. This 23 multiply by the quotient 3, and the product 69, subtract from 80, to the remainder 11, bring down the next period 21, makes 1121. Now 23 the divisor, in 112 goes 4, write 4 in the quotient, and on the right of 23, which now becomes 234; this 234 multiplied by the quotient 4, gives 936, and subtracting, there remains 187.

Now this quotient 34 added to the first root 200, makes 234, and if this 234 be cubed, it will be 12812904, which was the number first given, and therefore 234 is the true root required.

EXAMPLE II.

Extract the cube root out of 9239864750621 7, so that the root may consist of eight Figures or Places.

```
             92398647506217       (40000
             64                         3
             ---------------     ---------
  12,0000)   2839864750,6217      120000
             ---------------
       45)   236655395  (52
             225         400, &c.
             ----        ----
      425)   1165        452, &c
              904
             ----
```


Then

Then 452 cubed is 92345408. And the whole operation renewed, calling 452 the first root, will stand as follows;

```
           9239864750621 7     (45200
           92345408                 3
                                135600
1356,00) 532395062,17 (392621,727293
  45208) 392621,727293 (08,684, &c.
         361664
 452086) 3095772            45200
         2712516           08,684
4520868) 38325672        45208,684
         36166944        the root.
45208684) 215872893
          180834736
           35038157, &c.
```

In this example, when three places were found in the root, these were considered as the root of the three first periods; therefore 452 cubed gives 92345408, which used as the first cube of 4 was, *viz.* subtracted from the given number 9239864750621 7, leaves 532395062,17, this divided by thrice the root 452 with two cyphers annexed, (for the remaining two points over the number given,) *viz.* 45200, gives 392621,727293, which pointed as in the square, and the root 452 used as a divisor, *&c.* as before directed, gives 08,684, this added to the foregoing root, 45200, gives 45208,684 for the root required.

This

This method very seldom fails to give the root of any number true to three places at least, at the first operation; and if the second place in the root is a cypher, or an unit, four or five places may be obtain'd at the first operation.

But if a second operation is made, as in the foregoing example, eight or nine places in the root will be found: And if more accuracy be required, make a third operation, and this will give the root to 26 or 27 places; each operation tripling the figures found in the last root.

SECTION XI.

62. *Of* Characters *and their* Explanation.

IN the following sheets of this treatise, there are several characters and expressions used in order to shorten the work which are here explained.

I. Wherever is found this sign +, (*more*) it signifies that the number following the sign is to be added to the number going before it; thus 4 + 8 is read 4 more 8, and signifies, that 8 is to be added to 4.

II. This sign — (*less*) signifies that the number following it, is to be subtracted from the number going before it; thus 6 — 2, is read 6 less 2, and signifies, that 2 is to be taken from 6.

III. This sign × (*into*) signifies multiplication, and implies that the numbers this sign is between, are to be multiplied together; thus 4 × 9 imports, that 4 is to be multiplied by 9; and 2 × 3 × 6 × 5, signifies that 2 is to be multiplied by 3, and that product by 6, and this product by 5, and the like of any other.

IV. This ſign ÷ (*by*) ſignifies diviſion, and ſhews that the number going before the ſign, is to be divided by the number following it; thus 12 ÷ 4 implies, that 12 is to be divided by 4; but diviſion is moſt commonly expreſſed by ſetting down the dividend or number to be divided, and placing the diviſor or dividing number under it, with a line drawn between them, like a vulgar fraction; thus $\frac{12}{4}$ implies that 12 is to be divided by 4; and if 48,327 was to be divided by 2,15, expreſs it thus, $\frac{48,327,}{2,327,}$ &c.

V. This ſign = (*equal*) ſignifies that the numbers or expreſſions on each ſide thereof, are equal one to the other; thus 4 + 8 = 12, ſignifies that 8 added to 4 is equal to 12; and 6 − 2 = 4, implies that 2 taken from 6 leaves 4, or 6 leſſened by 2 is equal to 4; and 4 × 9 = 36, implies that 4 multiplied by 9, gives a product equal to 36; and $\frac{12}{4}$ = 3, ſignifies that 12 divided by 4, gives a quotient equal to 3, and the like of other expreſſions.

VI. The terms of proportions are expreſſed by certain points between the terms; thus 4 : 6 : : 10 : 15, and is read, as 4 is to 6, ſo is 10 to 15, ſo that the two points : between the two firſt terms is read, *is to*; the four points : : between the ſecond and third terms is read, *ſo is*, and the two points : between the third and fourth terms is read, *to*.

Take an example where all the forementioned characters are uſed: Suppoſe I buy 120 eggs at two a penny, and 120 more at three a penny, and ſell them again at five for two-pence; whether do I loſe or gain, and how much?

Now

Now $\frac{120}{2} = 60\ d.$ the price the firſt 120 coſt; and $\frac{120}{3} = 40\ d.$ the price that the ſecond 120 coſt; and $60 + 40 = 100$, the price 240 coſt. Then as 5 *egg.* : 2 *d.* : : 240 *egg.* : 96 *d.*; for $240 \times 2 = 480$, and $\frac{480}{5} = 96\ d.$ the price the eggs were ſold at; and $100\ d. - 96\ d. = 4\ d.$ the money loſt.

This ſign $\surd$, ſhews that the ſquare root is to be found, of thoſe quantities affected with the ſign.

Thus $\sqrt{40 \times \frac{18}{2}}$, ſhews that 40 is to be multiplied by $\frac{18}{2}$, and the ſquare root of the product is to be taken.

The character $\sqrt[3]{}$, denotes the cube root of the quantities following; thus $\sqrt[3]{4 + \frac{3}{2}}$ ſhews that the half of 3 is to be added to 4, and the cube root of the ſum to be taken.

When ſeveral terms, or numbers, are connected together by lines drawn either above or below them; it implies that the reſult of thoſe terms or numbers, ordered as their ſigns denote, is to be taken as one term or number.

Thus $4 \times \overline{2 + 5}$, ſhews that the ſum of 2 and 5, or 7, is to be multiplied by 4, and makes 28: But was it wrote thus $4 \times 2 + 5$; it would denote, that 4 was to be multiplied by 2, and the product 8, to be added to 5, making 13; which is very different from the former reſult.

 Again,

Again, $\overline{6 - \frac{\overline{3+5}\times 2}{4}} \times 7$, shews that the sum of 3 and 5, or 8, is to be multiplied by 2; and the product 16, to be divided by 4; and the quotient 4, to be subtracted from 6; and the remainder 2, to be multiplied by 7; so that the whole expression is equal to 14. But was it wrote thus, $6 - \frac{3+5\times 2}{4} \times 7$, the result would be very different; for it would denote, that the product of 5 by 2, or 10, was to be added to 3; and the sum 13, to be divided by 4, and the quotient $3\frac{1}{4}$ to be multiplied by 7; and the product $22\frac{3}{4}$, to be subtracted from 6. These things therefore should be carefully attended to, for the drawing of a line, or leaving it out, makes a wide difference in the meaning of an expression.

Several quantities, standing under a line, with a figure at the right-hand end, shews that the expression under that line, is to be raised to the power denoted by the index at the end of the line.

Thus $\overline{3 \times \frac{5}{4} + 6}^{\,2}$ denotes the second power of this expression, &c.

SEC.

SECTION XII.

Of DUODECIMALS.

63. *That scale of numeral notation, in which every superior place, is twelve times its next inferior, is called duodecimals.*

THIS way of conceiving the unit to be divided, is chiefly in uſe among workmen; and they uſe it only in caſting up the contents of their ſuperficial and ſolid works.

Artificers generally take the linear dimenſions of their work in feet, inches and parts.

And.	1 linear foot	=	12 linear inches.
	1 linear inch	=	8, or 12 linear parts.
Alſo.	1 ſquare foot	=	144 ſquare inches.
	1 ſquare inch	=	64, or 144 ſquare parts.
Again.	1 cubic foot	=	1728 cubic inches.
	1 cubic inch	=	512 or 1728 cubic parts.

The difference in the parts, ariſes from conſidering the inch as divided into 8 parts or 12 parts: For ſome workmen take their dimenſions in feet, inches, and half quarters, or eighth parts: Others take them in feet, inches and quarters, and reckon every quarter as 3 parts; ſo that theſe actually follow the duodecimal ſcale; altho' they ſeldom take notice of any other parts of an inch, than 3, 6, or 9, and give and take (as they call it) for the intermediate ones.

The operation of multiplying feet and inches, by feet and inches; or, feet, inches and parts, by feet, inches and parts, is commonly called *cross multiplication*.

As in decimals, the places descend by tens from the unit place to the right-hand, and are called primes, seconds, thirds, fourths, *&c.* so in duodecimals, the decrease is by twelves from the feet place to the right-hand, and are generally called feet, inches, parts, seconds, thirds, *&c.*

But as the terms, inches, parts, *&c.* are notions generally annexed to linear measure; and as the multiplication of these measures by one another produce denominations different from those in the multiplying factors; therefore, it will be more convenient to call the terms, inferior to feet, primes, seconds, thirds, fourths, *&c.* and these will equally suit linear, superficial and solid measures, just as well as the same names do in decimals, after any multiplication whatever.

And for distinction, let feet, be mark'd with (f;) primes, with (′;) seconds, with (″;) thirds, with (‴;) fourths, with (iv;) *&c.*

These marks may very properly be called the indices of the terms to which they belong; that is, they shew how many terms distant they are from the place of feet.

64. RULES *for multiplying duodecimally.*

1. Under the multiplicand, write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term; observing to carry an unit for every 12, from each lower denomination to its next superior.

3. In the same manner, multiply all the multiplicand by the primes in the multiplier; and write the result of each term, one place removed to the right-hand of those in the multiplicand.

4. Work in the same manner with the seconds in the multiplier, setting the result of each term, removed two places to the right-hand of those in the multiplicand.

The sum of these, gives the product required.

EXAMPLE I.

	f.	′	″				
Multiply	10 .	4 .	5			*the multiplicand*	= *A*
By	7 .	8 .	6			*the multiplier*	= *B*
Add	72 .	6 .	11 .	- .	-		= A × 7f.
	6 .	10 .	11 .	4 .	-		= A × 8′
		5 .	2 .	2 .	6		= A × 6″
	f.	′	″	‴	iv		
	79 .	11 .	0 .	6 .	6		= A × B = prod.

It amounts to the same, and is equally convenient, to begin first with the lowest name in the multiplier; observing to place the results right.

	f.	′	″				
Thus ex. 2. *mul.*	7 .	8 .	6			= A	
By	10 .	4 .	5			= B	
Add		3 .	2 .	6 .	6	= A × 5″	
	2 .	6 .	10 .	0 .		= A × 4′	
	77 .	1 .	0			= A × 10f.	
	f.	′	″	‴	iv		
	79 .	11 .	0 .	6 .	6	= A × B = prod.	

It will often happen, that the feet in the given dimensions are so many, that to multiply them by the lesser denominations, and to take a twelfth of their product, as above directed, will require some work to be done on a spare paper; to avoid which, observe the following,

65. RULE.

1. Divide the feet by 12, mentally reserving the quotient and remainder.

2. Multiply the reserved remainder by the lesser denominations, writing the over plus of 12 *s.* by each term, in their respective places.

3. Multiply the reserved quotient, by those lesser denominations, adding thereto, the 12 *s.* carried, and write the results in their proper places.

EX-

EXAMPLE III.

	f.	′	″	‴				f.	quo. rem.
Mul.	368 .	7 .	5			= *A*	Now	$\frac{368}{12}$	= 30 . 8
By	137 .	8 .	4			= *B*	And	$\frac{137}{12}$	= 11 . 5
Add	10 .	2 .	10 .	5 .	8	= A × 4″			
	245 .	8 .	11 .	4 .	-	= A × 8′			
	4 .	9 .	1 .	- .	-	= 137f. × 5″			
	79 .	11 .	-	- .	-	= 137f. × 7′			
	2576 .	-	-	-	-	= 368f. × 137f.			
	4784 .	-	-	-	-				
	f	′	″	‴	IV				
	50756 .	7 .	10 .	9 .	8				

EXAMPLE IV.

	f.	′	″	‴	IV			
Mul.	24 .	10 .	8 .	7 .	5			= *A*
By	9 .	4 .	6 .					= *B*
Add	224 .	0 .	5 .	6 .	9 .	- .	-	= A × 9f.
	8 .	3 .	6 .	10 .	5 .	8 .	-	= A × 4′
	1 .	0 .	5 .	4 .	3 .	8 .	6	= A × 6″
	f.	′	″	‴	IV	V	VI	
	233 .	4 .	5 .	9 .	6 .	4 .	6	= A × B = prod.

Ex-

Example 5. *Suppose the dimensions of a block of marble, were*

	f. ′ ″	
Length.	9 . 6 . 5	*required the solidity.*
Breadth.	4 . 7 . 8	
Depth.	2 . 10 . 3	

	f. ′ ″	
Multiply	9 . 6 . 5	= length = A
By	4 . 7 . 8	= breadth = B
	6 . 4 . 3 . 4	= A × 8″
	5 . 6 . 8 . 11 . -	= A × 7′
	38 . 1 . 8 . - . -	= A × 4f.
	f. ′ ″ ‴ IV	
And	44 . 2 . 9 . 2 . 4	= A × B
By	2 . 10 . 3	= depth = C
	88 . 5 . 6 . 4 . 8 . - . - . -	= A × B × 2f.
	36 . 10 . 3 . 7 . 11 . 4 . -	= A × B × 10′
	11 . 0 . 8 . 3 . 7 . 0	= A × B × 3″
	f ′ ″ ‴ IV V VI	
Gives	126 . 2 . 10 . 8 . 10 . 11 . 0	A×B×C = solidity.

That the several denominations arising from any multiplication (whether the factors have feet in them, or not) may be rightly estimated; the following rules will be found necessary.

66. RULE.

66. RULE.

Feet by feet, give feet.
Feet by primes, give primes.
Feet by seconds, give seconds,
&c.

Primes by primes, give seconds.
Primes by seconds, give thirds.
Primes by thirds, give fourths,
&c.

Seconds by seconds, give fourths.
Seconds by thirds, give fifths.
Seconds by fourths, give sixths,
&c.

Thirds by thirds, give sixths.
Thirds by fourths, give sevenths.
Thirds by fifths, give eighths,
&c.

In general thus:

When feet are concerned, the product is of the same denomination with the term multiplying the feet.

When feet are not concerned, the name of the product will be expressed by the sum of the indices of the two factors.

These rules will be sufficiently illustrated by the following example wrought at length.

Multiply

	f.	′	″	‴	IV	V	VI
Multiply	24	10	8	7	5		
By	9	4	6				
Th. 24f.×9f.=	216						
10′×9f.=		90					
8″×9f.=			72				
7‴×9f.=				63			
5IV×9f.=					45		
24f.×4′=		96					
10′×4′=			40				
8″×4′=				32			
7‴×4′=					28		
5IV×4′=						20	
24f.×6″=			144				
10′×6″=				60			
8″×6″=					48		
7‴×6″=						42	
5IV×6″=							30
ſum	216	186	256	155	121	62	30

	f	′	″	‴	IV	V	VI
Now 186′=	15	6					
256″=	1	9	4				
155‴=	-	1	0	11			
121IV=				10	1		
62V=					5	2	
30VI=						2	6
	233	4	5	9	6	4	6

	f.	′	″	‴	IV	V	VI
=	216	-	-	-	-	-	-
=	7	6	-	-	-	-	-
=	-	6	0	-	-	-	-
=	-	-	5	3	-	-	-
=	-	-	-	3	9	-	-
=	8	-	-	-	-	-	-
=	-	3	4	-	-	-	-
=	-	-	2	8	-	-	-
=	-	-	-	2	4	-	-
=	-	-	-	-	1	8	-
=	1	-	-	-	-	-	-
=	-	-	5	-	-	-	-
=	-	-	-	4	-	-	-
=	-	-	-	-	3	6	-
=	-	-	-	-	-	2	6
	233	4	5	9	6	4	6

Many examples, to exercise duodecimals, will be given in what follows.

The reader will readily see, that the numbers in each line in this page, are respectively equal to those in the opposite line in page 90.

A
TREATISE
OF
MENSURATION.

67. MENSURATION is the finding the contents of superficies and solids; by having their dimensions given in any sort of measure; and consists of three parts, *viz. lineal, superficial*, and *solid*.

68. *Lineal measure*, is the measuring of lengths, and is used in taking the dimensions of superficial and solid figures.

69. *Superficial measure*, is the finding the number of square inches, feet, yards, &c. in any plain or convex figure; as a floor, a wall, a globe, &c. by having the dimensions in lineal measure.

70. *Solid measure*, is the finding the number of cubic, or solid inches, feet, yards, &c. in those figures that have length, breadth, and depth; as a block of stone, marble, &c. by having the dimensions in lineal measure.

71. Of linear measure it may suffice to observe, that

12 inches - are -	1	foot.
3 feet - - - - - -	1	yard.
6 feet - - - - -	1	fathom.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet -	1	pole, perch or rod.
40 poles - - - - -	1	furlong.
8 furlongs - - - -	1	mile.

PART

PART I.

Of Superficial MEASURE.

Wherein will be shewn,

First, *Some useful definitions, and problems.*

Secondly, *The common methods of calculating used by artificers.* And

Thirdly, *Methods of computing the areas of various sorts of plane figures.*

SECTION I.

DEFINITIONS.

72. A Line is length without breadth; and is either right, when it is the shortest distance between two points; or curved, when it is not the shortest distance between two points.

73. A *superficies* is a figure which hath length and breadth, and is inclosed or contained between right or curved lines.

Note, One curved line may contain a space or superficies; but of right lines, less than three cannot contain a space.

74. When one line is inclined towards another line, in such a manner, as if either or both were continued, they would meet; then the opening of these lines is called *an Angle*, *Fig.* 1. *Pl.* 1.

75. When one line stands so on another, as to incline to neither side; but makes the angles on each

each side equal; each of those angles is called *a right one*; and the line so standing on the other, is called *a perpendicular*, to that whereon it stands. *Fig.* 2.

76. All three sided figures are called *triangles*, but admit of the following distinctions:

First, If the three sides are unequal, it is called *a scalene triangle*. *Fig.* 3.

Secondly, If the three sides are equal, it is called *an equilateral triangle*. *Fig.* 4.

Thirdly, If only two sides are equal, it is called *an Isosceles triangle*. *Fig.* 5.

Fourthly, If it has one right angle, it is called *a right-angled triangle*. *Fig.* 6.

77. All four sided figures are called *quadrilaterals*, but admit of the following distinctions:

First, When the four sides are equal; if the angles are right ones, it is called *a square*; *fig.* 7. but if the angles are not right ones, it is called *a rhombus*. *Fig.* 8.

Secondly, When the opposite sides only are equal; if the angles are right ones, it is called *a rectangle*; *fig.* 9. but if the angles are not right ones, it is called *a rhomboides*. *Fig.* 10.

Note, These four are called *parallelograms*, as having their opposite sides parallel or equidistant to each other; but all other four sided figures are called *trapeziums*. *Fig.* 11.

78. A circle is a plane figure, bounded by one curved line called the *circumference*; to which all right lines drawn from a certain point within the figure, called *its center*, are equal. *Fig.* 12.

79. The *diameter* of a circle is a right line drawn through the center, terminated at each end by the circumference, and divides the circle into the two equal parts, each called *a semicircle*. Half the diameter is called *a radius*. *Fig.* 12.

80. Every

80. Every circumference is ſuppoſed to be divided into 360 equal parts called *degrees*; each degree into 60 equal parts called *minutes*; each minute 60 equal parts called *ſeconds*, &c. And any part of a circumference is called an *arc*.

81. The *chord* of that arc, is a right line joining the ends of an arc: Or, a right line dividing a circle into two unequal parts called *ſegments*, is called a *chord*. *Fig.* 12.

82. If a chord cut a diameter at right angles, that part of the diameter lying between the chord and circumference, is called *a verſed ſine*, and is the height of the ſegment. *Fig.* 12.

83. A ſector is a figure contained under two radius's of a circle, and the arc included between thoſe radius's. *Fig.* 12.

84. A polygon is a figure contained under many ſides; if the ſides, and angles, are equal among themſelves, the figure is called a *regular polygon*; otherwiſe, an *irregular one*.

Note, A polygon is named according to its number of ſides, *viz.* If it has 5 ſides, it is called *a pentagon*; if 6 ſides, *a hexagon*; if 7, an *heptagon*; if 8, *an octagon*; if 9, *a nonagon*; if 10, *a decagon*; if 11, *an undecagon*; if 12, *a duodecagon*. See *Fig.* 13, 14, 15, 16, 17, 18, 19, 20.

85. In any quadrilateral, if a line be drawn to any two oppoſite angles, that line is called *a diagonal*. *Fig.* 21.

86. The altitude or height of any figure, is a perpendicular, let fall from the vertex of the figure to its baſe; that is, the line on which the figure is ſuppoſed to ſtand.

87. The area of any figure, is the ſuperficial content thereof.

SEC-

SECTION II.

AS there is ſometimes a neceſſity of letting fall a perpendicular, in order to come at the area of a figure; it is therefore convenient to know how to ſolve the following problems.

PROBLEM I.

88. *To biſect, or divide into two equal parts, the line A B.* (*Fig.* 22.)

CONSTRUCTION.

Set one foot of the compaſſes on the end B, open the other to any convenient diſtance greater than half AB, with that opening deſcribe the arch DE; ſet one foot on A, and with the ſame opening croſs the former arch in D and E; draw the line DE, and it will biſect the given line AB in the point C.

PROBLEM II.

89. *On any point* C *of a given line* AB, *to erect a perpendicular.* (*Fig.* 23.)

CONSTRUCTION.

On any convenient point as D, out of the given line, ſet one foot of the compaſſes, extend the other to the point C, with that extent, deſcribe a

circumference cutting AB in E; draw the diameter EDF; thro' the point C, and the extremity F of the diameter EF, draw the line CF, which will be perpendicular to the given line AB, and stand on the point C, as was required.

PROBLEM III.

90. *To let fall a perpendicular to any given line* AB, *from a given point* C, *above that line.* (*Fig.* 24.)

CONSTRUCTION.

Set one foot of the compasses on the point C, and with any convenient opening describe the arch DE, cutting the line AB in the points D, E; on the points D and E, with the same opening, describe arches below the line, to cross each other in F; lay a ruler by C and F, and draw the line CF, which will be perpendicular to the line AB, as was required.

SECTION III.

THE area of all right lin'd figures, may be obtained by the help of the two following propositions.

91. PROPOSITION I.

The length and breadth of a parallelogram being known; to find its area or superficial content.

RULE.

RULE.

Multiply the length by the breadth, the product will be the measure of the area required.

PROPOSITION II.

92. *Having given the base and perpendicular height of any right lined triangle; to find the area.*

RULE.

Multiply the base by half the perpendicular height; or the perpendicular height by half the base; and the product is the area.

By the help of these two propositions, workmen compute the areas of all right lined figures; and that by three different ways, *viz.*

First, By *aliquot parts*.

Secondly, By *decimals*.

Thirdly, By *duodecimals*.

But the latter of these are mostly used when the dimensions are taken in feet, inches, &c.

93. Different kind of works are computed by different measures, *viz.*

First, By the foot; as *glazing*.

Secondly, By the yard; as *painting*, *plaistering*, *paving*, &c.

Thirdly, By the square of 100 feet; as *flooring*, *partitioning*, *roofing*, *tyling*, &c.

Fourthly, By the rod of $16\frac{1}{2}$ feet, whose square is $272\frac{1}{4}$, by which *bricklayers* compute their work.

94. Land is best measured by the *Gunter's* chain, which is 4 poles long, and is divided into 100 links,

each being 7,92 inches; and as 40 poles in length, and 4 in breadth make a statute acre; therefore

625 squ. links = 1 pole

100000 - - = 160 = 1 acre = 4840 yards.

SECTION IV.

95. Of ARTIFICERS WORKS.

QUESTION I.

WHAT is the area or superficial content of a window, whose length is 14 feet 6 inches, and breadth 4 feet 9 inches?

This example is here exemplify'd by the three methods of operation.

First. By aliquot parts.		*Secondly.* By decimals.	*Thirdly.* By duodecimals.
In.			
6 $\frac{1}{2}$	14 - 6	14,5	14 - 6
	4	4,75	4 - 9
	58 - 0	725	58 - 0
	7 - 3	1015	10 - 10 - 6
$\frac{1}{2}$	3 - 7$\frac{1}{2}$	580	68 - 10 - 6
	68 - 10$\frac{1}{2}$	68,875	

The result, in each of these methods, is the same, which is, 68 square feet, and 10$\frac{1}{2}$ primes.

Altho' it is obvious, that linear feet drawn into linear feet, produce square feet; and inches by inches, produce square inches; and parts by parts, produce square parts: Yet feet, inches and parts, drawn into feet, inches and parts, produce other figures

figures and names besides square feet, square inches, and square parts, *viz.* one name or figure between the square feet and square inches, this multiplied by 12, give square inches: another name, between the square inches and square parts, this multiplied by 12, give square parts. See the following example.

f.	i.	p.				
39 .	10 .	7 .			= A	(See Art. 65.)
18 .	8 .	4 .			= B	
1 .	1 .	3 .	6 .	4	= A × 4p.	
26 .	7 .	0 .	8 .	-	= A × 8i.	
0 .	10 .	6 .	- .	-	= 18f. × 7p.	
15 .	0 .	- .	- .	-	= 18f. × 10i.	
702 .	- .	- .	- .	-	= 39f. × 18f.	
745 .	6 .	10 .	2 .	4	= A × B.	

Or. 745 . 72 + 10 . 24 + 4.
That is 745 sq. f. 82 sq. i. 28 sq. parts.

The names already given to the different denominations arising in duodecimal multiplication, are feet, primes, seconds, thirds, fourths, &c.; but it will not be improper in this place, to shew what figures those names really represent in superficial measure, produced by feet, inches and parts, drawn into feet, inches and parts.

1. The feet, are square feet.

2. The primes, are rectangles of a foot long and an inch wide.

3. The seconds, are either rectangles of a foot long, and 1 part wide; or, which is just the same, they are square inches.

4. The thirds, are rectangles of an inch long and 1 part wide.

5. The fourths, are square parts.

96. Of measurements by the foot square; as glazing, and masons flat work.

QUESTION II.

What will the glazing a triangular sky-light come to, at 10 d. per foot; supposing the base 12 feet 6 inches long, and the perpendicular height 16 feet 9 inches?

By duodecimals.

```
                          f.   i.
                          16 - 9 -     = the height.
                                6 - 3  = ½ the base.
                        --------------
                          4 - 2 - 3
 d.    d.               100 - 6
 6 ;   4 | ½, ⅓ |       --------------
                        104 - 8 - 3    = the area.
                        --------------
 In.   d.                52
 6 ½ | 10                34 - 8
 -----------              0 - 6⅞
 2 ⅓ |  5               --------------
        1⅔         20)  8,7 - 2⅞        Answer.
 -----------            --------------
        6⅔                4 7 2⅞        4 l. 7 s. 2⅞ d.
```

By decimals.

```
     16,75
      6,25
    ------
      8375
     3350       d.     £.
    10050      10=0,0416.
   --------
   104,6875
    6 140,0, the multiplier inverted. (Art 28.)
   --------
   9) 628
   --------
       697
      1047
     41875
   --------
    4,36197 £.
```

97. In the decimal work of this example, the multiplicand 104,6875 is first multiplied by 6, and then divided by 9, (*per* rule, p. 17.) because the 3 lines of product may stand together. And the same practice is to be understood in many of the following examples.

The area in the duodecimal work, is valued by aliquot parts.

QUESTION III.

There is a house with three tier of windows, three in a tier; the height of the first tier is 7 feet, 10 inches; of the second, 6 feet, 8 inches; of the third, 5 feet, 4 inches; and the breadth of each, 3 feet, 11 inches; what will the glazing come to at 14 d. per foot?

This question is best solved, by adding together the heights of the windows over each other, and multiplying the sum by 3, the number of rows; this gives a length equal to the 9 windows together.

By duodecimals.

$$\text{Add}\left\{\begin{array}{r} 7 - 10 \\ 6 - \ 8 \\ 5 - \ 4 \end{array}\right\}\text{the heights.}$$

```
                    19 - 10
Multiply by               3  heights.
                    59 -  6 = heights toget.
Mult.                 3 - 11 = breadth.
                    54 -  6 -  6
                   178 -  6
                   233 -  0 -  6 = area

P.    d.          d.
6 | 1/4 | 14      | 2 | 1/6 | 233
  |     | 0½      |   |     |  38 - 10
                                    8/9
               20) 27,1 - 10½
             £. 13 - 11 - 10½ Answer.
```

By decimals.

(Art. 97.)

```
        59,5 = heights together.
        3,916 = breadth.
    9)  3570
        3966
        595
       5355
      1785
     233,0416 and 14 = 0,0583
      3850,0
    9)699
       776
      18643
     116521
  £. 13,5940
```

Glaziers

Glaziers generally meaſure their work to a quarter of an inch; and never make any allowances for round or oval windows, but always meaſure them to the greateſt length; for there is more trouble in cutting the glaſs to thoſe ſhapes, than the value of the glaſs omitted.

QUESTION IV.

What is a marble ſlab worth, whoſe length is 5 feet 7 inches; and breadth 1 foot 10 inches, at 6 s. per foot?

		By duodecimals.	By decimals.
		5 - 7 - 0	5,583
		1 - 10	1,83
		4 - 7 - 10	9) 16750
s.		5 - 7	18611
5	1/4	10 - 2 - 10	44666
1	1/5	2 - 10	55833
		10	10,2361
		1 - 5	0,3 = 6 s.
		3 - 01 - 5	3,07083
		Anſwer.	
		3 l. 1 s 5 d.	

i.p.		s.
2.0	1/6	6
0.6	1/4	1-0
0.3	1/2	3
0.1	1/3	1 1/2
		1/2
		1-5

98. Of

98. Of measurements by the yard square: as paviours, painters, plaisterers and joiners.

In these works, the dimensions are taken in feet; and the result given in square yards, each of 9 square feet.

Hence divide the area found in square feet by 9, the quotient will be the number of square yards required.

QUESTION V.

What will the paving a court of a rectangular form come to, at 3 s. 2 d. per yard; supposing the length 27 feet, 10 inches, and the breadth 14 feet, 9 inches.

By duodecimals.

```
                     27 - 10
                          14 -  9
                    ---------------
                     20 - 10 -  6
                     11 -  8
                    378
                    ----------------
                 9) 410 -  6 -  6                s.  d.
                    ----------------
            s.d.     45 -  5 -  6 - 6 at 3 - 2
                    ----------------
            2.0. | 1/10 | 45
f. i. p.   s. d.        |-------
                   1.0. | 1/2  |  4 - 10
3 0.0 | 1/3 | 3 - 2     0.2. | 1/6  |  2 -  5
      |     |-------              7 -  6
1.0.0 | 1/3 | 1 - 0½              1 - 10¾
1.0.0 | 1/1 |     4         ----------------
0.6.0 | 1/2 |     4¾        £. 7 - 4 - 4¾ Answer.
0.0.6 | 1/12|     2
      |     |     0
            ---------
            1 - 10¾
```

By

By decimals.

27,83
14,75
13916
19483
11133
27833
9) 410,5416 s.d. £.
45,6157 at 3.2 = 0,1583
3851,0
(See Art. 97.) 9) 137
152
3649
22808
45616
£. 7,2225

QUESTION VI.

One has paved a rectangular court-yard 42 feet, 9 inches in front; and 68 feet, 6 inches in depth: And in this he laid a foot-way the depth of the court; of 5 feet, 6 inches in breadth: The foot-way is laid with purbeck stone, at 3 s. 6 d. per yard, and the rest with pebbles, at 3 s. per yard; what will the whole come to?

Find the value of the whole Court at 3 *s.* and the foot-way at 6 *d.* these values added together, will give the whole cost.

```
 68 -  6      | s. |      | yds.  f.   i.  p.      s.| f. i. p.|     | s.  d.
      42 - 9  | 2  | 1/10 |  325 - 3 -  4 - 6  at 3 | 3 - -   | 1/3 | 3 - 0
 -----------  | 1  | 1/2  |  ----------------------  |    4 -  | 1/9 | 1
 51 -  4 - 6  |    |      |   32 - 10                |      6  | 1/8 |      1¼
 21 -  0      |    |      |   16 -  5                |         |     | ---------
 136          |    |      |          1 -  1¼         |         |     | 1 -  1¼
 272          |    |      |  ---------------------   |--------------------------
 ------------ |    |      |  £. 48 - 16 - 1¼         | f. i.   |     |
9) 2928 - 4 - 6 | d. |    |  ---------------------   | 3 - -   | 1/3 | - 6 0
 ------------ |    |      |  yds.  f.   i.  d.       | 3 - -   | 1/1 |     2
  325 - 3-4-6 | 6  | 1/40 |   41 -  7 - 9 at 6       | 1 - -   | 1/3 |     2
 The court's area.        |  ---------------------   | 0 9 -   |     |     0½
 ------------ |    |      |       1 - 0 - 6          |         |     |     0½
 68 -  6      |    |      |               5          |         |     | -------
        5 - 6 |    |      |  ---------------------   |         |     | 0 - 5
 ------------ |    |      |  £.  1 - 0 - 11          |--------------------------
 34 -  3 - 0  |
 342 -  6     |
 ------------ |
9) 376 - 9    |
 ------------ |
 41 -  7 - 9  |
 The foot-way's area.
```

	l.	*s.*	*d.*
The whole court at 3 *s.* is	48 —	16 —	1¼
The foot-way at 6 *d.* is -	1 —	—	11
The whole cost ——— £.	49 —	17 —	¼

QUES.

QUESTION VII.

What will the plaistering a ceiling at 10 *d. per yard come to; supposing the length* 21 *feet,* 8 *inches, and the breadth* 14 *feet,* 10 *inches?*

By duodecimals.

			21 - 8		
			14 - 10		
			18 - 0 - 8		
			9 - 4		
			294		
d.	*d.*	9)	321 - 4 - 8		*d.*
6 *and* 4	$\frac{1}{2}$, $\frac{1}{3}$		35 - 6 - 4 - 8	at 10	
			17 - 6		
			11 - 8		
			$6\frac{2}{4}$		
		20)	29 - $8\frac{2}{4}$		
			1 - 9 $8\frac{2}{4}$		

Answer, 1 *l.* 9 *s.* $8\frac{2}{4}$ *d.*

f. i.		*d.*
3.0	$\frac{1}{3}$	10
0	$\frac{1}{1}$	$3\frac{1}{4}$
4	$\frac{1}{9}$	$3\frac{1}{1}$
		$\frac{1}{2}$
		$6\frac{3}{4}$

By

By decimals.

```
        1 4,8 3
          2 1,6
9)    8 9 0 0        (See Art. 97.)
        9 8 8 8
      1 4 8 3
    2 9 6 6 6
9)  3 2 1 8 4
    3 5 7 0 9 8, &c. at 10 d. = 0,0416 £.
      6 1 4 0,0
9)    2 1 4
        2 3 8
        3 5 7
    1 4 2 8 3
    1,4 8 7 8
```

Plaisterers works are principally of two kinds, namely, *first*, works lath'd and plaister'd, which are call'd *ceiling*. *Secondly*, works *rendered*, which is of two kinds, *viz.* upon brick walls, or between quarters in the partition between rooms.

In measuring *rendering* upon brick walls, there are no deductions made; but in measuring *rendering* between *quarters*, there is commonly a fifth part of the whole area deducted: But when *rendering* between *quarters* is whited or colour'd, there is commonly a fourth or fifth part added to the whole area, for the sides of the *quarters* and *braces*, &c.

Note, Make proper deductions for doors, windows, &c.

QUES-

QUESTION VIII.

There is a quantity of partitioning that measures 234 feet, 8 inches about, and 14 feet, 6 inches high; but is reduced between quarters: The lathing and plaistering will be 8 d. per yard, and the whiting 2 d. per yard; what will the whole come to?

By duodecimals.		By decimals.
234 - 8		234,6
14 - 6		14,5
117 - 4 - 0		11733
9 - 4		93866
3276		234666
9) 3402 - 8		9) 3402,6
5) 378 - 0 - 8	the area in yards.	5) 378,0740
75 - 5 - 6	this $\frac{1}{5}$ part deduct.	75,6148
302 - 4 - 2	rem. the ar. plaister'd.	302,4592
378 - 0 - 8	to the area in yards.	378,0740
75 - 5 - 6	add the $\frac{1}{5}$ part.	75,6148
453 - 6 - 2	gives the ar. whited.	453,6888

302,

302,4592	453,6888	
8 *d.* = ,03 £.	2 *d.* = ,0083 £.	
10,081973	1512296	10,081973
	3629510 4	3,6446333 6
	3,6446333 6	J. 13,7266063 6

Anſwer, 13 *l.* 14 *s.* 6 *d.* the whole coſt.

99. In theſe two operations, inſtead of multiplying by 3, and dividing by 9, (as directed, p. 17.) take $\frac{1}{3}$ of the multiplicand, which is exactly the ſame, but more expeditious.

QUESTION IX.

Suppoſe a room that was painted at 8 *d. per yard, meaſures as follows: The height, (taking in the cornice and mouldings) is* 11 *feet,* 7 *inches; the girt or compaſs,* 74 *feet,* 10 *inches; the door* 7 *feet,* 6 *inches, by* 3 *feet,* 9 *inches; five window-ſhutters, each* 6 *feet,* 8 *inches, by* 3 *feet,* 4 *inches; the breaks in the windows,* 14 *inches deep, and* 8 *feet high; the chimney* 6 *feet,* 9 *inches, by* 5 *feet; a cloſet the height of the room,* $3\frac{1}{2}$ *feet deep, and* $4\frac{1}{4}$ *feet in front, with ſhelving, at* 22 *feet,* 6 *inches, by* 10 *inches; the ſhutters, door, and ſhelves, are coloured on both ſides: What will the whole come to?*

7438

7 4,8 3 × 1 1,5 8 3 =	— —	866,8194	The area of the whole room.
6,6 × 3,3 × 5 =	— —	111,1111	The area of the shutters once.
(8 + 8 + 3,3 + 3,3 =) 22,6 × 1,16 × 5 =		132,2,222	The breaks in the windows.
7,5 × 3,7 5 =	— —	28,125	The door once.
(3,5 × 2 + 4,75 × 2 =) 16,5 × 11,583 =		191,125	The closet's area.
2 2,5 × 8 3 × 2 =	— —	37,5	The area of the Shelves.
The sum of the areas.	— — —	1366,9027	Out of which deduct.
6,7 5 × 5 =	— — — —	33,75	The chimney's area.
Remains	— — — —	1333,1527	The area of the whole work.

Then $\frac{1333,1527}{9}$ = 148,128, &c. *yds.* then if 1 *yd.* : ,03 £. :: 148,128 *yd.* : 4,9376 £. &c. = 4 £. 18 *s.* 9 *d.*

Painters

Painters take their dimensions with a string, and measure from the top of the cornice to the floor, girting the string over all the mouldings and swelling pannels: and in measuring of doors, they account the height and breadth of the door so much more, as in the thickness of the stuff; it being reasonable they should be paid for all places whereon their colour is laid. Their price they generally proportion according to the number of times they lay their colour on.

Note, There must always be made deductions for Chimnies, Casements, *&c.* if any within the dimensions taken.

QUESTION X.

What will the wainscoting a room come to at 6 *s. per square yard; supposing the height of the room (taking in the cornice and mouldings) is* 12 *feet* 6 *inches, and the compass is* 83 *feet* 8 *inches; the window shutters each* 7 *feet* 8 *inches, by* 3 *feet* 6 *inches, and the door* 7 *feet by* 3 *feet* 6 *inches; the shutters and door being worked on both sides, is reckoned work and half work?*

```
  83 -  8
       12 - 6
  ------------
  41 - 10 - 0
1004 -  6
-----------
1045 - 10 - 0 = { room's area, including
                { the shut. and do. once.
-----------
   7 -  8
        3 - 6
  ------------
   3 - 10 - 0
  23 -  0
  ------------
  26 - 10 - 0
            3
  ------------
  80 -  6 - 0 = three shutters once.
  ------------
   3 -  6
   7 -  9
  ------------
  24 -  6 - 0 = door once.
  ------------
2) 105 - 0 = shutters and door.
  ------------
   52 - 6 = half the shutters and door.
```

```
           f.     i.
         1045 - 10 = room
           52 -  6 = ½ shut. and door.
         -----------
       9) 1098 -  4 = whole work.
         -----------
 s. d.   | yds.  f.  i.
 4.0 | ⅕ | 122 - 0 - 4 at 6 s. per yard.
         -----------
         |  l.   s.  d.
in. 2.0 | ½ |  24 - 8 - 0
4 | 1/27 | 6.0 |  12 - 4 - 0
  | 0.2½ |        0 - 2½
         -----------
       l. 36 - 12 - 2½ the answer.
```

Joiners

Joiners measure their work in height with a string, and their length or compass upon the floor as painters do; for they say, they ought to measure where their plane touches, therefore they take the cornice and mouldings into the height of the room.

Note, They only take the cornice and mouldings in with the height of the room, when they are struck with a *horse plane*, (as they call it) but if they are wrought by hand, then they are paid so much *per foot* running measure.

All stuff an inch and half thick and under, wrought on both sides, is by them reckoned at work and half work; but stuff of greater thickness wrought on both sides, is valued at double work.

They make deduction for all vacancies that fall within their work; and *window-boards*, *sofite-boards*, *cheeks*, &c. are measured by themselves.

100. Of measurements by the square. As flooring, partitioning, roofing, tyling, &c.

In these works, the dimensions are taken by a rod of ten feet; and therefore the result is in squares of 100 square feet each.

Hence, divide the area found in square feet by 100, the quotient will be the number of squares required.

QUESTION XI.

Suppose a house of three stories, beside the ground-floor, was to be floor'd at 6 *l.* 19 *s. per square; the house measures* 20 *feet* 8 *inches, by* 16 *feet* 9 *inches; there are* 7 *fire-places, whose measures are; two, each of* 6 *feet, by* 4 *feet* 6 *inches; two other, each of* 6 *feet, by* 5 *feet* 4 *inches; and two, each of* 5 *feet* 8 *inches, by* 4 *feet* 8 *inches; and the seventh,* 5 *feet* 2 *inches, by* 4 *feet; and the well-hole for the stairs, is* 10 *feet* 6 *inches, by* 8 *feet* 9 *inches; what will the whole come to?*

```
4 - - 6      5 - - 4      5 - - 8
      6            6          4 - - 8
27 - - 0     32 - - 0     3 - - 9 - - 4
       2            2     22 - - 8
54           64           26 - - 5 - - 4
                                       2
      20 - 8              52 - - 10 - - 8
        16 - 9
      15 - 6 - 0
     330 - 8
     346 - 2
           4
    1384 - 8 the area of the four floors.
     559 - 0 - 8 the area of the deduct.
100) 825 - 7 - 4 the area of the work.
f.  f.   i.  p.  £.  s.
8 . 25 - 7 - - 4 at 6 - - 10 per square.
             l.   s.   d.
Answer, 53 - - 13 - - 3¼
```

```
5 - - 2     10 - 6
      4          8 - 9
20 - - 8    7 - 10 - 6
            84
            91 - 10 - 6
                     4 the number of floors.
           367 - 6 - 0 the well hole.
            54 - 0 - 0 the first chimney.
            64 - 0 - 0 the second chimney.
            52 - 10 - 8 the third chimney.
            20 - 8 - 0 the fourth chimney.
           559 - 0 - 8 the whole deductions.
```

s.	sqrs.			l. s. d.
10 - ½	8	25 f.	¼	6 - 10 - 0
	6	6 i.	1/50	1 - 12 - 6
	48	1	1/6	7¾
	4	4 p.	⅓	1¼
	1 - 13 - 3¼			
	£. 53 - 13 - 3¼			1 - 13 - 3¼

In

In flooring they deduct the hearth-stone, except it has a border round it; and then the hearth is measured in with the floor.

Question XII.

In 173 feet 10 inches in length, and 10 feet 7 inches in height of partitioning; how many squares?

By duodecimals.	By decimals.
173 - 10	1 7 3,8 3
10 - 7	3 8 5,0 1 } breadth inverted.
101 - 4 - 10	9) 5 2 (See 97.)
1738 - 4	5 8
100) 1839 - 8 - 10	1 3 9 0
f. *f.* *i.* *p.*	8 6 9 1
18 — 39 — 8 — 10	1 7 3 8 3 3 *sqrs.*
	100) 1 8 3 9,7 2 (18,3972

In *roofing*, *tyling* and *slating*, it is customary to reckon the flat and half of any building within the walls, to be the measure of the roof of that building; when the said roof is of a *true pitch*.

Note, All roofs are said to be of a *true pitch*, when the rafters are $\frac{3}{4}$ of the breadth of the building.

If the roof is more or less than *true pitch*, they measure from one side to the other with a rod or string.

QUESTION XIII.

If a house measure within the walls 52 feet, 8 inches in length, and 30 feet, 6 inches in breadth; and the roof be of a true pitch; what will it cost roofing at 10 s. 6 d. per square?

By duodecimals. | **By decimals.**

```
30 - - 6 — the breadth of the building ——— 30,5
15 - - 3 —— the half breadth —— ——      15,25
45 - - 9 —— the breadth of the roof — — 45,75
     52 - - 8                             52,6
30 - - 6 - - 0                         9) 27450
39 - - 0                                  30500
90                                         9150
2225                                      22875
100) 2409 - - 6                     100) 2409,5
                                         24,095 sqrs.
                                           ,525
                                         120475
                                         48190
                                        120475
                                       12,649875
```

```
s. d.  |      | sq.  f.  i.      s.  d.
10.0   | 1/2  | 24 - 9 - 6 at 10 - 6 per square.
0.6    | 1/20 | 12 - 0          | f.       f.  |            | s.   d.
       |      | - - - 12        | 5 and 4      | 1/20, 1/25 | 10 - 6
       |      | - - - - 11 3/4  |          1   |            | — 6 1/4
       |      | 12 - 12 - 11 3/4|          6   | 1/6        | — 5
       |      |                 |              |            | — 1/2
       |      |                 |              |            | — 11 3/4
```

Answ. 12 l. 12 s. 11 3/4 d.

There

There are other works about a building done by the carpenter, that are measured by the foot *running measure*; as *cornices*, *doors*, *and cases*; *window-frames*, *lintels*, *guttering*, *sommers*, *skirt-boards*, &c.

In the measuring of roofing for workmanship alone; they generally deduct the holes for chimney shafts and skylights, if they are any thing considerable.

But measuring for work and materials, they commonly measure in all *skylights*, *lutheren lights*, and *holes* for the *chimney shafts*, for their trouble and waste of *stuff*; excepting such *skylights* as exceed nine or ten feet in area.

QUESTION XIV.

What will the tyling a barn cost, at 25 *s.* 6 *d. per square; the length being* 43 *feet*, 10 *inches*, *and breadth* 27 *feet*, 5 *inches on the flat; the eaves boards projecting* 16 *inches on each side?*

By

By duodecimals.

```
            27 - 5 ——— the breadth
            13 - 8 - 6 — the half breadth
1 - 4 × 2 =  2 - 8 —— the eaves boards doubled —
            43 - 9 - 6
               43 - 10 - 0
            36 - 5 - 11 - 0
             1 - 9 - 6          s.   d.
            32 - 3              25 - 6 = 1,275
           129
          1720
     100) 1919 - 6 - 5
          f.   f.   i.  p.      s.   d.
          19 - 19 - 6   5 at 25 - 6 per square.
```

```
s. d.                      f. f.               s.   d.
 5    | 1/4 | 19           10  4 | 1/10,1/25 | 25 -  6
 0  6 | 1/10|  4 = 15           5 |    1/2   |  2 -  0 1/2
              9 - 6        6 i.   |          |  1 -  3 1/4
              4 - 11 1/2          |    1/8   |  1 -  0 1/4
Ans. l. 24 - 09 - 05 1/2          |          |     1 1/2
                                              4 - 11 1/2
```

By decimals.

```
     2 7,4 1 6 6
     1 3,7 0 8 3
       2,6 6 6 6
     4 3,7 9 1 6
         3 8,3 4  length inverted.
  9) 1 3 1 4
       1 4 6 0
     3 5 0 3 3
   1 3 1 3 7 5
 1 7 5 1 6 6 6                 sqrs.
100) 1 9 1 9,5 3 4 (1 9,1 9 5 3 4
                          5 7 2,1
                    1 9 1 9 5 3
                      3 8 3 9 0
                      1 3 4 3 7
                          9 6 0
                    2 4,4 7 4
```

Note, In angles formed in a roof, running from the ridge to the eaves, that angle of the roof, which bends inwards, is called a *valley*; and the angle bending outwards, is called a *Hip*; and in tiling and ſlating, it is common to add the length of the *Vallies* (meaſured from the ridge to the eaves) to the content in feet; ſometimes the *Hips* are added.

In ſlating, it is common to reckon the breadth of the roof 2 or 3 inches broader than what it meaſures, becauſe the firſt row is almoſt cover'd by the ſecond; this is done ſometimes when a roof is tiled.

Note, Skylights and chimney ſhafts are deducted, but they ſeldom deduct luthern lights (or garret windows on the roof) for the covering of ſuch, they reckon it equal to the hole in the roof.

101. Of meaſurements by the rod, as brick-work.

In this work, the dimenſions are eſtimated by a rod of $16\frac{1}{2}$ feet; and therefore the reſult is in ſquare rods of $272\frac{1}{4}$ ſquare feet each.

Hence (for practice) divide the area found in ſquare feet by 272, the quotient will be the rods required.

Note, Bricklayers always compute or value their work at the rate of a brick and an half thick; and if the thickneſs of the wall happen to be more or leſs than ſuch, it muſt be reduced to that thickneſs as follows:

Multiply the area of the wall, by the number of half bricks the thickneſs of the wall is of; divide the product by 3, and it gives the area.

QUES-

QUESTION XV.

How many square rods are there in a wall 62½ feet long, 14 feet, 8 inches high, and 2½ bricks thick?

By duodecimals.

62 - 6
14 - 8
41 - 8 - 0
875 - 0
272) 916 - 8 (3 - 100 - 8   r. f. i.
5
3) 16 - 231 - 4
5 - 167 - 9 - 4

r. f. i. p.
Answ. 5 - 167 - 9 - 4

By decimals.

62,5
14,6
9) 3750
4166
8750
272) 91666 (3,37, &c. rods.
5
3) 16,85
5,616, &c.

In measuring of brick-work, they are very careful in regard to their allowances; for one foot square in the front is commonly worth sixpence.

In great buildings, they often deduct the timbers laid in the walls; but this is only when the workmanship is very good; for in general, it is allow'd in; because they reckon the time they wait on the carpenter, together with the bedding of those timbers in mortar, is equal to the brick-work that would supply the timbers place.

QUESTION XVI.

Suppose the side walls of an house to be 28 *feet,* 10 *inches in length; and the height of the roof from the ground,* 53 *feet,* 8 *inches; and the gable (or triangular part at top) to rise* 42 *course of bricks (reckoning* 4 *course to a foot.) Now* 20 *feet high is* 2½ *bricks thick;* 20 *feet more, at* 2 *bricks thick;* 15 *feet* 8 *inches more, at* 1½ *brick thick; and the gable of* 1 *brick thick; what will the whole work come to at* 5 *l.* 16 *s. per rod?*

$\frac{42}{4} = 10{,}5$ the height of the gable; its half is 5,25 *feet*.

```
   28,83          28,83          28,83
      20             20           15,6
  -------        -------     -----------
   576,6          576,6     9) 17300
       5              4         1922
  -------        -------        43250
3) 28833      3) 23066
  -------        -------       -------
   961,1          768,8         451,72
    f.   £.        f.    £.
As 272 : 5,8 :: 2282,638 : 48,671

          £.     s.    d.
Answer,   48 -- 13 -- 5¼
```

```
        28,83
         5,25
      -------
        14416
        57666
      1441666
      -------
      151,375
            2
      -------
   3) 302,75
      100,916  at 1   }
      451,722  at 1½  } Bricks
      768,888  at 2   } thick.
      961,111  at 2½  }
      --------
      2282,638
```


In

In all buildings, the thickneſs of the walls generally decreaſe as they riſe; and it is uſual to ſet off half a brick at each decreaſe: The thickneſs is commonly ſet off on the inſide, and that in a place where a floor will come, ſo that the ſet-off is thereby hid.

It is common to build from a baſe 4 courſe of bricks high, and which projects two or three inches on each ſide of the wall.

The different thickneſſes are meaſured ſeparate, and reduced each to a brick and an half thick; and then added together.

To meaſure a chimney ſtanding by itſelf, without any party-wall being adjoined; girt it about for the length, and reckon the height of the ſtory for the breadth: the thickneſs muſt be the ſame the Jaumbs are of, provided that the chimney be wrought upright from the mantle-tree to the ceiling; not deducting any thing for the vacancy between the floor (or hearth) and the mantle-tree; becauſe of the gathering of the breaſt and wings, to make room for the hearth in the next ſtory.

If the chimney-back be a party-wall, and the wall be meaſured by itſelf; take the depth of the two Jaumbs, and the length of the breaſt for a length; and the height of the ſtory is the breadth, at the ſame thickneſs your Jaumbs were of.

To meaſure chimney-ſhafts, or that part which appears above the roof; girt them with a line, about the leaſt place for the length; and take the height for the breadth; and if they be four inches thick, ſet down the thickneſs at one brick-work; but if they be wrought nine inches thick, (as ſometimes they are, when they ſtand alone and high, above the roof) reckon the thickneſs at a brick and half, in conſideration of the plaiſtering (call'd *pargeting*) and trouble of ſcaffolding.

It

It is customary in most places to allow double measure for chimnies.

More Examples, *to exercise the foregoing* Propositions.

102. The areas of parallelograms and triangles, being divided by one of their dimensions, will give the other dimension.

Question XVII.

What difference is there between a floor 48 feet long, and 30 feet broad; and two others, each of half the dimensions?

Now 48 × 30 = 1440; and 24 × 15 × 2 = 720; but 720 is half of 1440.

Therefore any plane figure, whose linear dimensions are double to the dimensions of another like figure, contains four times the area.

Question XVIII.

From a mahogany plank 26 inches broad; a yard and an half in area is to be saw'd off; what distance from the end must the line be struck?

Now $1\frac{1}{2}$ yards area = $13\frac{1}{2}$ feet area.
And 26 inches = 2,16 feet.

 Then

Then $\frac{13,5}{2,16} = 6,23$ feet, the distance from the end the line must be struck.

QUESTION XIX.

A joist is $8\frac{1}{2}$ inches deep, and $3\frac{1}{2}$ broad; I want a scantling just as big again, that shall be $4\frac{3}{4}$ inches broad; what will be the other dimension?

Now $8,5 \times 3,5 \times 2 = 59,5$; and $\frac{59,5}{4,75} =$ 12,52 inches deep, the answer.

QUESTION XX.

I have a girder 19 inches by 13; but one that has but a quarter of the timber in it, so it be 10 inches deep, will serve my purpose; how broad must it be?

Now $19 \times 13 = 247$, and $\frac{247}{4} = 61,75$.

Then $\frac{61,75}{10} = 6,175$ inches, the breadth.

QUESTION XXI.

A roof is 24 feet 8 inches, by 14 feet 6 inches on the flat; and covered with lead at 8 ℔ to the foot; what will it come to at 18 s. per Cwt.

Now

Now 24,6 × 14,5 = 357,6 f. the area. (Art. 24.)
And 8 ℔ = 0,071428 *Cwt.*

Then as 1 f. : 0,071428 *Cwt.* : : 357,6 f. : 25,5473 *Cwt.*

Then as 1 *Cwt.* : 0,9 £. : : 25,5473 *Cwt.* : 22,992 £.

Or 22 £. 19 *s.* 10½ *d.* is the cost.

QUESTION XXII.

A plumber made for a mason a leaden cistern, every foot square whereof weigh'd 19 ℔ at 19 s. per Cwt. the dimensions were 81 inches long, 46 inches deep, and 36 inches broad; with three stays across it, of the same strength, and each 18 inches deep: For which the mason was to pave with purbeck stone, at 7½ per foot, a square pavement, that should just balance the cost of the cistern; what must the side thereof be?

Now 81 + 36 = 117, which × by 2 gives 234 = the length of the two sides and two ends; or the girth of the cistern; then 234 × 46 = 10764 inches, the area of the sides and end: And 81 × 36 = 2916 inches, the area of the bottom.

Again, 36, the length of one stay, × by 18 the depth, gives 648 inches, which × 3, gives 1944 inches, the area of the three stays. Then 10764 + 2916 + 1944 = 15624 inches, the area of the whole cistern and stays.

Note, 19 ℔ =, 0,1696, *&c. Cwt.*

Then, as 144 *sq. in.* : ,1696 *Cwt.* : : 15624 *sq. in.* : 18,4016 *Cwt.* The shortest way to work this proportion is, divide the third term by the first term, gives 108,5, and × the second term by this

quote, gives the fourth term, which is the weight of the cistern.

Then if 1 *Cwt.* : ,95 £. : : 18,4016 *Cwt.* 17,48152 £. = 17 £. 9 *s.* $7\frac{1}{2}$ the expence that the cistern does amount to.

Therefore, as ,03125 £. : 1 f. : : 17,48152 £. : 559,408 f. the area of the square; and extracting the square root, will give 23,65 f. &c. the side thereof.

QUESTION XXIII.

There is a fleight of iron railing 42 feet long, the bars whereof are $\frac{3}{4}$ of an inch square, and the whole weight is $12\frac{3}{4}$ Cwt. which is to be changed for some that are inch and $\frac{4}{5}$ strong, exchange at $4\frac{1}{2}$ d. per lb; what will the whole come to?

Now as the strength of the bars are expressed by the areas of the ends.

	str.		*str.*		*Cwt.*		*Cwt.*
Therefore	$\overline{0{,}75}^2$	:	$\overline{1{,}8}^2$	: :	12,75	:	73,44.
	lb		*d.*		lb		*d.*
And	1	:	$4\frac{1}{2}$	: :	112	:	504 = 2 £.

	Cwt.		£.		*Cwt.*		£.
Then	1	:	2,1	: :	73,44	:	154,124.

So the expence will be 154 £. 2 *s.* 6 *d.*

QUES-

QUESTION XXIV.

A triangular field that is 1777,7 links in the base; and 900 links in the perpendicular; brings in 36 £. per annum: How much is it lett for per acre?

Now $1777,7 \times \frac{900}{2} = 800000$ square links the area.

And $\frac{800000}{100000} = 8$ acres.

Then $\left(\frac{36}{8} = 4,5\text{ £.} =\right)$ 4 £. 10 *s.* the rent *per* acre.

QUESTION XXV.

If a man can mow the grass standing on a square rod in one minute: How long will he be in mowing a meadow whose length is 728 yards, and breadth 358 yards, supposing him to work 14 hours per day?

Now $778 \times 358 = 260624$ yards.

Or 8615,669 square rods.

Then 8615,669 min. = 10 days, 3 h. 3 min.

SECTION V.

Of the areas of divers right-lined figures.

103. PROPOSITION III.

Given the three ſides of a triangle; to find the area.

RULE.

FROM half the ſum of the three ſides, ſubtract each ſide ſeverally; let the half ſum, and the three differences, be multiplied continually; the ſquare root of the product will be the area required.

EXAMPLE I.

If the ſides of a triangular field are 15, 14, 13, *perches; what is the content of that field?*

Now ſum of ſides $= (15 + 14 + 13 =)$ 42; $\frac{1}{2}$ ſum $= 21$;

And $21 - 15 = 6$; $21 - 14 = 7$; $21 - 13 = 8$:

Then $21 \times 6 \times 7 \times 8 = 7056$.

And the ſquare root of 7056 is 84 ſquare perches the area required.

Ex-

EXAMPLE II.

A field of a triangular form, whose sides are 380, 420 and 765 yards, letts for 55 s. per acre; how much does the whole bring in per annum?

Now $380 + 420 + 765 = 1565$.

And $\frac{1565}{2} = 782,5$ yards, the half sum of the three sides.

And $782,5 - 380 = 402,5$, the first difference.

$782,5 - 420 = 362,5$, the second differ.

$782,5 - 765 = 17,5$, the third difference.

Also $782,5 \times 402,5 \times 362,5 \times 17,5 = 9998003710,9375$, whose square root is 14699,034 square yards, &c. which divided by 1840 (the square yards in one acre) gives 9,2353 acres.

Then, as 1 acre : 2,75 £. : : 9,2353 : 25,397 £.

Or, 25 £. 7 s. $11\frac{1}{2}$ d. is the yearly rent of the field.

104. PROPOSITION IV.

Two sides of a right angle triangle being given, to find the other side.

CASE I.

The two perpendicular sides (or legs) being given, to find the other side, or hypothenuse. v. *Fig.* 25.

RULE.

RULE.

Square each side, add the squares together, and the square root of this sum gives the *hypothenuse* required.

CASE II.

If the *hypothenuse* and one *leg* be given, to find the other *leg*.

RULE.

From the square of the *hypothenuse*, subtract the square of the given *leg*; the square root of the remainder, gives the *leg* required.

EXAMPLE I.

Wanted the length of a shoar, that strutting 14 *feet from the upright of a building, may support a jaumb* $20\frac{1}{2}$ *feet from the ground?*

Now $20,5 \times 20,5 = 420,25$; and $14 \times = 196$; then $420,25 + 196 = 616,25$; whose square root is 24,82 feet, &c. the length of the *shoar* required.

EXAMPLE II.

A line of 380 *feet will reach from the top of a precipice that stands close by a brook-side, to the opposite bank: And the precipice is known to be* 128 *feet high; how broad is the brook?*

Now $380 \times 380 = 144400$; and $128 \times 128 = 16384$; then $144400 - 16384 = 128016$; whose square

square root is 357,79 feet, &c. the breadth of the brook required.

Example III.

A ladder 52½ feet long, may be so placed in a street, that it shall reach a window 29 feet from the ground on one side; and by turning the ladder over, (without removing the foot,) it will touch a moulding 40 feet from the ground on the other side; how broad is the street?

First, 52,5 × 52,5 = 2756,25; and 29 × 29 = 841; then 2756,25—841 = 1915,25; whose square root is 43,76 feet, the breadth between the ladder and building the first situation.

Secondly, 40 × 40 = 1600; and 2756,25 — 1600 = 1156,25; whose square root is 34 feet, the breadth between the ladder and building, the second situation.

Then 43,76 + 34 = 77,76 feet, the breadth of the street required.

PROPOSITION V.

To find the area of any trapezium, the diagonal, and perpendiculars let fall thereon from the opposite angles being given.

RULES.

1. Multiply the given diagonal by the sum of the perpendiculars, and take half the product.

2. Or, Multiply the diagonal by half the sum of the perpendiculars.

3. Or,

3. Or, Multiply half the diagonal by the sum of the perpendiculars.

4. Or, Find the area of each triangle, and take their sum.

Either of these rules will give the area required.

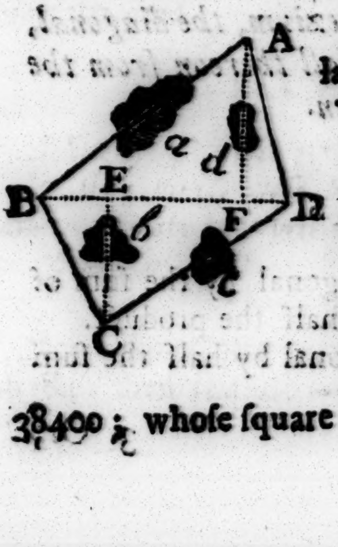
EXAMPLE I.

What is the area of a trapezium, whose diagonal is 34 feet, 9 inches, and the sum of the perpendiculars is 28 feet, 6 inches?

Now half of 28f. 6 inch. is 14f. 3 in. = 14,25f.
And 34,75 × 14,25 = 495,1875 feet, the area.

EXAMPLE II.

In the quadrilateral meadow ABCD, *the four ponds* a, b, c, d, *prevent the measuring of any other lines than the diagonal* BD, *which is* 378 *yards; the side* BC=220 *yards, and the side* AD=265 *yards: But it is known, that the perpendicular* CE *will fall* 100 *yards from* B; *and the perpendicular* AF *falls* 70 *yards from* D: *Required the area of that meadow?*



First find the perpendiculars CE, AF, by prop. IV.
Thus 265 × 265 = 70225.
And 70 × 70 = 4900.
And 70225 — 4900 = 65325, whose square root is 255,58, &c. = ~~CE.~~ AF
Again, 220 × 220 = 48400.
And 100 × 100 = 10000.
Then 48400 — 10000 = 38400; whose square root is 195,95, &c. = ~~AF.~~ CE

And

And $\frac{255,58 + 195,95}{2} = 225,76$ the half sum of the perpendiculars.

Then $\frac{225,76 \times 378}{4840} = 17,632$ acres.

106. PROPOSITION VI.

To find the area of a trapezium having two sides parallel; those sides and their perpendicular distance being known.

RULES.

Multiply half the sum of the parallel sides by the breadth.

Or, Multiply the sum of the parallel sides, by half the breadth.

Or, Multiply the sum of the parallel sides by the breadth, and take half the product.

Either of which will give the area.

EXAMPLE.

What is the area of a trapezium, having two parallel sides, the one 180 links, the other 120; and whose breadth is 80 links?

Now $\overline{180 + 120} \times \frac{80}{2} = 12000$ square links.

And 625 square links $=$ 1 square pole.

Then 625 : 1 :: 12000 : 19,2 poles the area.

107. PRO-

107. PROPOSITION VII.

To find the area of right-lin'd irregular figures.

RULE.

Divide the figure into as many triangles as may be, by drawing lines from any one angle to all the other angles; and the sum of the areas of the several triangles will be the area of the figure. *Fig.* 29.

But the lines (frequently) may be better disposed, than by drawing them all from the same angle, *viz.* by dividing the figure into trapeziums, as many as may be, and leaving as few single triangles as possible; for the area of a trapezium is more expeditiously obtained, than the areas of the two triangles which compose the trapezium, found separately. *Fig.* 30.

All right-lined figures may be divided into as many triangles (without any of the dividing lines cutting each other) as the figure has sides, abating two.

EXAMPLE.

Suppose a meadow, an irregular figure of 8 sides, which is lett for 31 shillings per acre; upon the mensuration is divided into three trapeziums: In the first, the diagonal is 4 chains and 24 links; and the sum of the perpendiculars is 3 chains, 67 links; in the second, the diagonal is 7 chains, 43 links, and the sum of the perpendiculars is 5 chains, 38 links; in the third, the diagonal is 6 chains, 78 links, and the sum of the perpendiculars is 4 chains, 84 links; what will the whole bring in per annum? Fig. 30.

Now

Now 4 *C.* 24 *L.* = 424 *L.* and 3 *C.* 67 *L.* = 367 *L.*

Then $367 \times \frac{424}{2} = 77804$ *L.* the area of the first trapezium.

Also 7 *C.* 43 *L.* = 743 *L.* and 5 *C.* 38 *L.* = 538 *L.*

Then $743 \times \frac{538}{2} = 199867$ *L.* the area of the sec.

Again, 6 *C.* 78 *L.* = 678 *L.* and 4 *C.* 84 *L.* = 484 *L.*

And $678 \times \frac{484}{2} = 164076$ *L.* the area of the third.

Then $77804 + 199867 + 164076 = 441747$ *L.* $= 4,41747$ *acres*, which at 34 *s.* = 1,7 *l.* comes to 7,509699 £. = 7 £. 10 *s.* 2¼ *d.* the answer.

---

108. PROPOSITION VIII.

The side of a regular polygon, its perpendicular distance from the centre, and the number of sides being given; to find the area of that polygon.

RULE.

Multiply half the sum of the sides by the perpendicular.

Or, Multiply the sum of the sides by half the perpendicular.

Or, Multiply the sum of the sides by the perpendicular, and take half the product.

Either of these rules will give the area.

Ex-

EXAMPLE.

What is the area of a regular pentagon, whose side is 25 yards; and the perpendicular let fall from the center to one of its sides, is 17,2 yards.

$$\left.\begin{array}{l}\text{Then } \dfrac{25\times5}{2}\times17{,}2\\[1ex] \text{Or } 25\times5\times\dfrac{17{,}2}{2}\\[1ex] \text{Or } \dfrac{25\times5\times17{,}2}{2}\end{array}\right\} = 1075 \text{ square yards, is the area sought.}$$

The subject of regular polygons, will be more fully treated hereafter.

SECTION VI.

109. *Of a circle and its parts.*

THERE is no figure that affords a greater number of useful properties than the circle; but the chief of these depend on knowing the relation which the diameter has to the circumference.

The determining of this proportion has exercised the thoughts of several mathematicians, both ancients and moderns. Some, in expectation of discovering the proportion accurately, but herein they always failed; for it has been demonstrated, that

that the relation of the diameter to the circumference, is not to be expressed in known measure: But others, more knowing, have contented themselves in approximating this relation.

110. *Archimedes*'s proportion is in whole numbers, *viz.* of 7 to 22; that is, supposing the diameter of a circle to be 7; he found the circumference of the same circle to be 22 nearly; for 22 is too much.

111. *Metius* found out a proportion of 113 to 355; that is, supposing the diameter of a circle to be 113, he found the circumference of the same circle, 355 nearly; but it is also too great, tho' much nearer the truth than *Archimedes*.

112. Van *Culen*'s proportion is much more accurate than any before his time; for he, supposing the diameter of a circle to be 1, found, (with prodigious labour and trouble) the circumference to be 3,141592653.5897932384.6264338327.950288 which was then thought so great a work, and so curious a performance, that the numbers were cut on his tomb-stone in the church-yard of St. *Peter*'s at *Leyden* (as related by some.)

113. But by methods which the moderns are possessed of, the same thing (and many others of a like intricate nature) may be perform'd with abundantly less labour and trouble; as is sufficiently shewn in most of the late elementary treatises; particularly in the *Synopsis palmariorum matheseos* of the late most excellent mathematician, Mr *Jones*; wherein is given the late Mr. *Machin*'s series for the rectification of the circle; and thereby the proportion of the diameter to the circumference, true

true to 100 places; that is, supposing (as Van *Culen* did) the diameter to be 1, the circumference is found to be 3,141592653.5897932384.626 4338327.9502884197.1693993751.0582097494. 4592307816.4062862089.9862803482.53421170 67.9 + true to 100 places of figures, and that computed in very little time, compared with the methods used by the ancients.

The proportion of Van *Culen* is now most commonly used, and seems the best adapted to practice; as it saves the trouble of division: but then it must be observ'd, that it is thought accurate enough, in most practical affairs to use the number 3,1416, the excess, being not $\frac{1}{10000}$ part of the unit.

The relation between the diameter and circumference being once established, it is easy to deduce a great variety of useful properties, relating not only to the circle, but also, to its circumscribing and inscrib'd regular polygons; some of the most applicable to practical measuring, are contain'd in the ensuing pages; wherein, the computations are considerably facilitated by the application of the factors in the following tables.

314. A TABLE of useful factors, wherein p represents the circumference of a circle whose diameter is 1.

p	=	3,1415927	$p\sqrt{2}$	=	4,4428829
$2p$	=	6,2831853	$p\sqrt{\frac{1}{2}}$	=	2,2214415
$4p$	=	12,5663706	$\frac{1}{p}\sqrt{\frac{1}{2}}$	=	0,2250791
$\frac{p}{2}$	=	1,5707963	$\frac{1}{p}\sqrt{2}$	=	0,4501581
$\frac{p}{4}$	=	0,7853982	$\sqrt{p}$	=	1,7724538
$\frac{4p}{3}$	=	4,1887902	$\frac{1}{2}\sqrt{p}$	=	0,8862269
$\frac{p}{6}$	=	0,5235988	$2\sqrt{p}$	=	3,5449076
$\frac{p}{8}$	=	0,3926991	$\sqrt{\frac{p}{2}}$	=	1,25331414
$\frac{p}{12}$	=	0,2617994	$\sqrt{\frac{2}{p}}$	=	0,7978846
$\frac{p}{360}$	=	0,0087267	$\sqrt{\frac{1}{p}}$	=	0,56418958
$\frac{1}{p}$	=	0,3183099	$2\sqrt{\frac{1}{p}}$	=	1,1283792
$\frac{2}{p}$	=	0,6366197	$\frac{1}{2}\sqrt{\frac{1}{p}}$	=	0,2820948
$\frac{4}{p}$	=	1,2732395	pp	=	9,8696044
$\frac{1}{4p}$	=	0,0795775	$\frac{1}{pp}$	=	0,1013212
$\sqrt{2}$	=	1,4142136	$\frac{1}{2pp}$	=	0,0506605
$\sqrt{\frac{1}{2}}$	=	0,7071068	$\frac{1}{6pp}$	=	0,0168868

In

In regular polygons, whose number of sides do
1st. The side of the polygon.
2d. The radius of the circumscribing circle.
Then any one of these four quantities being given.

115. TAB. I. When the side of the polygon is 1.

Numb. of sides.	Rad. of the circu. circ.	Rad. of the inscri. circ.	The area.
3	0,5773503	0,2886751	0,4330127
4	0,7071068	0,5000000	1,0000000
5	0,8506508	0,6881910	1,7204774
6	1,0000000	0,8660254	2,5980762
7	1,1523825	1,0382617	3,6339124
8	1,3065630	1,2071068	4,8284271
9	1,4619022	1,3737387	6,1818242
10	1,6186340	1,5388418	7,6942088
11	1,7747329	1,7028437	9,3656404
12	1,9318516	1,8660254	11,1961524

116. TAB. II. When the radius of the circum. circle is 1.

Numb. of sides.	Length of the side.	Rad of the inscri. circ.	The area.
3	1,7320508	0,5000000	1,2990381
4	1,4142136	0,7071068	2,0000000
5	1,1755705	0,8090170	2,3776412
6	1,0000000	0,8660254	2,5980762
7	0,8677674	0,9009689	2,7364102
8	0,7653668	0,9238795	2,8284271
9	0,6840403	0,9396926	2,8925437
10	0,6180340	0,9510565	2,9389263
11	0,5634651	0,9594931	2,9735250
12	0,5176381	0,9659259	3,0000000

Put N = tabular number of any column

not

not exceed 12; if there be conſidered,
3d. The radius of the inſcrib'd circle.
4th. The area of the polygon.
The other three are readily found by theſe Tables.

117. Tab. III. When the radius of the inſcrib. circle is 1.

Numb. of ſides.	Length of the ſide.	Rad. of the circum. cir.	The area.
3	3,4641016	2,0000000	5,1961524
4	2,0000000	1,4142236	4,0000000
5	1,4530851	1,2360680	3,6327128
6	1,1547005	1,1547005	3,4641016
7	0,9631491	1,1099160	3,3710222
8	0,8284271	1,0823919	3,3137084
9	0,7279405	1,0641776	3,2757315
10	0,6498394	1,0514622	3,2491970
11	0,5872521	1,0422172	3,2298913
12	0,5358984	1,0352760	3,2153904

118. Tab. IV. When the area is 1.

Numb. of ſides.	Length of the ſide.	Rad. of the circum. cir.	Rad of the incrib. circ.
3	1,5196716	0,8773827	0,4386912
4	1,0000000	0,7071068	0,5000000
5	0,7623870	0,6485251	0,5246678
6	0,6204033	0,6204033	0,5372849
7	0,5245813	0,6045183	0,5446520
8	0,4550899	0,5946034	0,5493420
9	0,4021996	0,5879764	0,5525172
10	0,3605106	0,5833184	0,5547687
11	0,3267617	0,5799148	0,5564242
12	0,2988585	0 5773503	0,5576775

in either of theſe four tables.

119. PROPOSITION IX.

The side (S) *of a regular polygon being given,* *Fig.* 27. 28.

I. *To find the radius* (R) *of the circumscribing circle.*

RULE. Multiply the given side by N, and the product is the radius sought.

Ex. *What is the radius of a circle that can circumscribe a regular octagon whose side is* 12?

Here $N = 1,3065630$, found in Tab. I. (115.)
Then $1,3065630 \times 12 = 15,678756 = R$.

II. *To find the radius* (r) *of the inscrib'd circle?*

RULE. N, multiplied by the given side, gives r.

Ex. *If* S *is* 12, *what is* r?

Here $N = 1,2071068$, found in Tab. I. (115.)
Then $1,2071068 \times 12 = 14,4852816 = r$.

III. *To find* (A) *the area.*

RULE. N, multiplied by the square of the given side, gives the area.

Ex-

Ex. *If* S *is* 12, *what is* A?

Here N = 4,8284271, found in Tab. I. (115.)
Then 4,8284271 × 12 × 12 = 695,2935024 = A.

120. PROPOSITION X.

The radius (R) *of a circle circumscribing a regular polygon being given.* Fig. 27. 28.

I. *To find* (S) *the side of that polygon.*

Rule. N, multiplied by the given radius, gives S.

Ex. *If the radius of a circle is* 12; *required the side of the inscribed regular octagon?*

Here N = 0,7653668, found in Tab. II.
Then 0,7653668 × 12 = 9,1844016 = S.

II. *To find* (r) *the radius of a circle inscrib'd in that polygon.*

Rule. N, multiplied by R, gives r.

Ex. *If R is* 12; *what is r?*

Here N = 0,9238795, found in Tab. II.
Then 0,9238795 × 12 = 11,086554 = r.

III. *To find* (A) *the area of that polygon.*

RULE. N, multiplied by the square of the given radius, gives the area sought.

Ex. *If R = 12; what is A?*

Here N = 2,8284271, found in Tab. II.
Then 2,8284271 × 12 × 12 = 407,2935024 = A.

121. PROPOSITION XI.

The radius (r) *of a circle inscrib'd in a regular polygon, being given.* Fig. 27. 28.

I. *To find* (S) *the length of the side of that polygon.*
RULE. N, multiplied by the given radius, gives S.

Ex. *What is the side of a regular octagon, circumscribing a circle whose radius is 12?*

Here N = 0,8284271, found in Tab. III.
Then 0,8284271 × 12 = 9,9411252 = S.

II. *To find* (R) *the radius of a circle that will circumscribe the polygon.*

RULE. N, multiplied by r, gives R.

Ex.

Ex. *If r = 12; what is R?*

Here N = 1,0823919, found in Tab. III.
Then 1,0823919 × 12 = 12,9887028 = R.

III. *To find* (A) *the area of that polygon.*

Rule. N, multiplied by the ſquare of r, gives A.

Ex. *If r = 12; what is A?*

Here N = 3,3137084, found in Tab. III.
Then 3,3137084 × 12 × 12 = 477,1740096 = A.

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122. PROPOSITION XII.

The area (A) *of a regular polygon being given.* Fig. 27. 28.

I. *To find* (S) *the length of its ſide.*

Rule. N, multiplied by the ſquare root of the given area, the product is the ſide ſought.

Ex. *What is the ſide of a regular octagon, whoſe area is* 144?

Here N = 0,4550899, found in Tab. IV.
Then 0,4550899 × $\sqrt{144}$ = 5,4610788 = S.

 II. *To*

II. *To find* (R) *the radius of a circle circumscribing that polygon.*

Rule. N, multiplied by the square root of A, gives R.

Ex. *If* A = 144; *what is* R?

Here N = 0,59496034, found in Tab. IV.
Then $0{,}5946034 \times \sqrt{144} = 7{,}1352408 = R$.

III. *To find* (r) *the radius of a circle inscrib'd in that polygon?*

Rule. N, multiplied by the square root of A gives r.

Ex. *If* A = 144; *what is* r?

Here N = 0,549342, found in Tab. IV.
Then $0{,}549342 \times \sqrt{144} = 6{,}592104 = r$.

---

123. PROPOSITION XIII.

The diameter of a circle being known.

I. *To find the circumference.*

Rule. Multiply the diameter by 3,1416 (= *p*) and the product is the circumference.

Ex.

Ex. *If the diameter is 12; what is the circumf.*

Then $(12 \times 3{,}1416 =)$ 37,6992 is the circumf.

II. *To find the area.*

Rule. Multiply the ſquare of the diameter by 0,7854 ($\frac{1}{4}p$) and the product is the area.

Ex. *If the diameter is 12, what is the area?*

Then $(12 \times 12 \times 0{,}7854 =) 113{,}0976$ is the area.

Or, The radius, multiplied by half the circumference, gives the area of the circle.

Thus $\frac{37{,}6992}{2} \times 6 = 113{,}0976$.

Again, the ſquare of the diameter multiplied by 0,392699 $\left(= \frac{1}{8} p\right)$ gives the area of a ſemicircle.

III. *To find the ſide of a ſquare equal in area to the circle.*

Rule. Multiply the diameter by 0,886217 $(= \frac{1}{2}\sqrt{p})$ and the product is the ſide of the ſquare.

Ex. *If the diameter is 12; what is the ſide of the equal ſquare?*

Then $(12 \times 0{,}8862 =)$ 10,6344 is the ſide of the ſquare.

 124. PRO-

124. PROPOSITION XIV.

The circumference of a circle being known.

I. *To find the diameter.*

RULE. Multiply the circumference by 0,31831, $\left(=\frac{1}{p}\right)$ and the product is the diameter.

Ex. *Suppose the circumference is 12; required the diameter?*

Then $(12 \times 0{,}31831 =)$ 3,81972 is the diameter.

II. *To find the area.*

RULE. Multiply the square of the circumference by 0,0795776 $\left(=\frac{1}{4p}\right)$ and the product is the area.

Ex. *Suppose the circum. is 12; required the area?*

Then $(12 \times 12 \times 0{,}07958 =)$ 11,45952 is the area.

III. *To find the side of an equal square.*

RULE. Multiply the circumference by 0,282095, $\left(=\frac{1}{2}\sqrt{\frac{1}{p}}\right)$ and the product is the side of the equal square.

Ex-

Ex. *Suppose the circumference is 12; required the side of a square, of equal area to that circle?*

Then $(12 \times 0,2821 =) 3,3852$ is the side required.

125. PROPOSITION XV.

The area of a circle being known.

I. *To find the diameter.*

RULE. Multiply the square root of the area by $1,12837$, $\left(=2\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter.

Ex. *When the area is 12; what is the diameter?*
Then $(\sqrt{12} \times 1,12837 =) 3,90877$ is the diameter.

II. *To find the circumference.*

RULE. Multiply the square root of the area by $3,5449$, $(=2\sqrt{p})$ and the product will be the circumference.

Ex. *When the area is 12; what is the circumf.*
Then $(\sqrt{12} \times 3,5449 =) 12,3798$ is the circumf.

III. *To find the side of a square of equal area.*

RULE. The square root of the given area, will be the side of the square required.

Ex. *When the area is 12, what is the side of the equal square?*

Then ($\sqrt{12}=$) 3,4641 is the side required.

126. PROPOSITION XVI.

The side of a square, or its area, being known.

I. *To find the diameter of a circle of equal area.*

RULE. Multiply the side of the square, by 1,12837 $\left(=2\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter sought.

Ex. *What is the diameter of that circle, equal in area to a square whose side is 12?*

Then ($12\times1,12837=$) 13,54044 is the diameter required.

II. *To find the circumference of an equal circle.*

RULE. Multiply the side of the square by 3,5449, $(=2\sqrt{p})$ and the product will be the circumference sought.

Ex. *What is the circumference of that circle, whose area is equal to a square wherein the side is 12?*

Then ($12\times3,5449=$) 42,5388 is the circumf. required.

III. *To*

III. *To find the side of a square, that may be inscrib'd in that circle of equal area to the given square.*

Rule. Multiply the given side by 0,797884, $\left(=\sqrt{\frac{2}{p}}\right)$ and the product is the side of the square sought.

Ex. *What is the side of that square, which may be inscrib'd in a circle of equal area to a square whose side is 12?*

Then $(12 \times 0{,}797884 =)$ 9,574608 is the side required.

IV. *To find the area of a square, that may be inscrib'd in a circle of equal area to the given square?*

Rule. Multiply the square of the given side by 0,63662, $\left(=\frac{2}{p}\right)$ and the product is the area of the square required.

Ex. *What is the area of that square, which may be inscrib'd in a circle of equal area to a square whose side is 12?*

Then $(12 \times 12 \times 0{,}63662 =)$ 91,67328 is the area required.

127. PROPOSITION XVII.

The radius (CA) *of a circle, and the chord* (AB) *of an arc thereof being known: To find the versed sine* (DE) *of half that arc.* Fig. 3.

Rule. From the ſquare of the radius, take the ſquare of half the chord; the ſquare root of the remainder, ſubtracted from the radius, leaves the verſed ſine.

Or $DE = CA - \sqrt{\overline{CA}^2 - \overline{AE}^2}$.

Ex. *In a circle whoſe radius is 25: What is the verſed ſine of that arc, the chord of whoſe double is 48?*

Now $25 \times 25 - 24 \times 24 = 49$; whoſe ſquare root is 7.

Then $(25 - 7 =)$ 18 is the verſed ſine required.

128. PROPOSITION XVIII.

The radius (CA) *of a circle, and the verſed ſine* (ED) *of an arc* (BD) *thereof being known:* Fig. 33.

I. *To find the chord* (AB) *of twice that arc.*

Rule. From twice the radius, take the verſed ſine; multiply the remainder by the verſed ſine; then will twice the ſquare root of the product give the chord of twice the arc.

Or $BA = 2\sqrt{\overline{\overline{DF} - \overline{DE}} \times \overline{DE}}$.

Ex. *In a circle whoſe radius is 25; what is the chord of twice the arc, whoſe verſed ſine is 18?*

Then $\sqrt{\overline{2 \times 25 - 18} \times 18} \times 2 =)$ 48 the chord required.

II. To

II. *To find the chord* (BD) *of that arc.*

RULE. Multiply twice the radius by the versed sine, and the square root of the product will be the chord of the arc.

Or $BD = \sqrt{DF \times DE}$.

EX. *If the radius of a circle is* 25, *and the versed sine of an arc thereof, is* 18: *What is the chord of that arc?*

Then $(\sqrt{2 \times 25 \times 18} =)$ 30 is the chord required.

129. PROPOSITION XIX.

The chord (AB) *of any circular arc, and the versed sine* (ED) *of half that arc being known.*

I. *To find the radius* (CA) *of the circle.* Fig. 33.

RULE. To the square of half the chord, add the square of the versed sine; divide the sum by twice the versed sine, and the quotient is the radius required.

Or $CA = \overline{\overline{BE}^2 + \overline{DE}^2} \div 2\,DE$.

EX. *If the chord* AB $= 48$, *and the versed sine* DE $= 18$; *what is the radius of the circle?*

Then $\left(\frac{\frac{48}{2} \times \frac{48}{2} + 18 \times 18}{2 \times 18} =\right)$ 25 is the radius required.

Note,

Note, The distance of the chord from the centre, is found by subtracting the versed sine from the radius.

II. *To find the chord* (BD) *of half the arc.*

RULE. To the square of half the chord, add the square of the versed sine; and the square root of the sum, will be the chord of half the arc.

Or $BD = \sqrt{\overline{BE}^2 + \overline{DE}^2}$.

Ex. *If the chord is* 48, *and the versed sine is* 18; *what is the chord of half the arc?*

Then ($\sqrt{24 \times 24 + 18 \times 18} =$) 30 is the chord of half the arc.

130. PROPOSITION XX.

To find the length of a circular arc (BDA) Fig. 33.

I. *When the chord* (AB) *of that arc, and the chord* (BD) *of its half are known.*

RULE. From 8 times the chord of half the arc, subtract the chord of the whole arc; and $\frac{1}{3}$ of the remainder, will be the length of the arc nearly.

Arc $ADB = \frac{1}{3} \times \overline{8\,BD - AB}$.

Ex *If the chord of the arc is* 48, *and the chord of half the arc is* 30; *what is the length of the arc?*

Then

Then $\left(\frac{30 \times 8 - 48}{3} = \right)$ 64 is the length of the arc.

II. *When the chord* (AB) *of an arc,* (ADB) *and the versed sine* (DE) *of half the arc are known.*

RULE. Find the diameter (by Case I. Prop. XIX.) divide $\frac{2}{3}$ of the versed sine, by the diameter lessened by $\frac{82}{100}$ of the versed sine; the quotient added to 1, and the sum multiplied by the chord, will give the length of the arc very near.

$$\text{Arc ADB} = \overline{\frac{\frac{2}{3}DE}{DF - \frac{82}{100}DE} + 1} \times AB.$$

EX. *If the chord of an arc is* 48, *and the versed sine of half the arc is* 18: *What is the length of that arc?*

Now $\left(\frac{24 \times 24 + 18 \times 18}{18} = \right)$ 50 is the diameter.

And $(50 - \frac{82}{100} \times 18 =)$ 35,24 is the divisor.

Then $\left(1 + \frac{\frac{2}{3} \times 18}{35,24} \times 48 = \right)$ 64,31496 is the length of the arc.

III. *When the diameter* (DF) *of a circle, and the arc* (ADB) *in degrees are known.*

RULE. Multiply the degrees in the arc, by the diameter of the circle; the product multiplied by 0,0087267 $\left(= \frac{p}{360}\right)$ will shew the length of the arc in that kind of measure the diameter is of.

EX.

Ex. *In a circle whose diameter is 50 feet; what is the length of an arc of 147 degrees, 29 minutes?*

Now 147 deg. 29 min. = 147,483 degrees.

Then ($147,483 \times 50 \times 0,0087267 =$)64,352 feet, is the length of the arc required.

Note, The arc in degrees may be easily found, by having the diameter of the circle, and the length of the arc, found by either of the foregoing Rules; Thus.

Divide the number 114,59132 ($= \frac{360}{p}$) by the diameter, the quotient multiplied by the length of the arc, gives the arc in degrees.

131. PROPOSITION XXI.

To find the area (CADB) *of a circular sector.* Fig. 34. 35.

I. *When the radius* (CA) *of the circle, and the length of the sectoral arc* (ADB) *are known.*

RULE. Multiply the radius by half the given arc, and the product will be the area of the sector.

Ex. *In a circle whose radius is 25; required the area of a sector on the arc whose length is 64,352?*

Then ($\frac{64,352}{2} \times 25 =$)804,4 is the area requir'd.

II. *When the radius of the circle, and the degrees in the sectoral arc known.*

RULE.

RULE. Multiply the given degrees, by the square of the radius; the product multiplied by 0,0087267 $\left(=\frac{p}{360}\right)$ will give the area of the sector.

EX. *In a circle whose radius is 25 feet: What is the area of a sector on an arc of 147 degrees, 29 minutes?*

Now 147 deg. 29 min. $=$ 147,483.
Th. (147,483 × 25 × 25 × 0,0087267 =) 804,4017 is the area of the sector required.

132. PROPOSITION XXII.

To find the area (ADBA $=$ A) *of a circular segment, whose height,* (DE $= v$) *and chord* (AB $= b$) *of its arc are known.* Fig. 36.

RULE I. Multiply the height by 0,626; to the square of the product, add the square of half the chord: Multiply twice the square root of the sum, by two thirds of the height, and the product is the area.

$$\text{Or } A = \tfrac{2}{3}v \times 2\sqrt{\overline{0{,}626v}|^2 + \overline{\tfrac{1}{2}b}|^2}$$

EX. *What is the area of that circular segment whose height is* 18; *and the chord of whose arc is* 48?

Now 18 × 0,626 = 11,268.
And 11,268 × 11,268 + $\frac{48}{2} \times \frac{48}{2}$ = 702,967824, whose square root is 26,5135.

Then $\left(26{,}5135 \times 2 \times \frac{2 \times 18}{3} =\right)$ 636,324 is the area of the segment.

RULE.

RULE II. *Find the chord* (AD$=b$) *of half the arc.* (By Case II, Prop. XIX.) Then,

To the square of the chord, add the square of the height; to twice the square root of the sum, add the chord of half the arc; multiply the sum by $\frac{4}{15}$ of the height, and the product will give the area.

Or $A = \overline{2\sqrt{bb+vv}+b} \times \frac{4}{15}v$.

Ex. *In a circular segment whose height is* 18, *and the chord of the arc is* 48: *What is the area?*

Now ($\sqrt{24 \times 24 + 18 \times 18} =$) 30, is the chord (AD) of half the arc.

And $\sqrt{48 \times 48 + 18 \times 18} = 51,26402$.

Then ($\overline{51,26402 \times 2 + 30} \times 18 \times \frac{4}{15} =$) 636,13459 is the area of the segment.

RULE III. Find the diameter (by Case I. Prop. XIX.) divide 32 times the height, by 80 times the diameter lessened by 15 times the height; take the quotient from the number $\frac{4}{3}$; multiply the remainder by the square root of the product of the diameter and height; this product multiplied by the height, will give the area of the segment.

$$\text{Or } A = v\sqrt{dv} \times \overline{\tfrac{4}{3} - \frac{32v}{80d - 15v}}$$

Ex. *What is the area of a circular segment, the height being* 18; *and the chord of the arc* 48?

Now $\left(\frac{24 \times 24 + 18 \times 18}{18}\right)$ 50 is the diameter.

And

And $18 \times 32 = 576$ is the dividend.
And $30 \times 80 - 18 \times 15 = 3730$ is the divisor.

Then $\frac{576}{3730} = 0{,}15442$, &c.

Therefore $(\overline{\tfrac{2}{3} - 0{,}15442} \times \sqrt{\overline{50 \times 18}} \times 18 =)$ 636,6814 is the area of the segment.

RULE IV. Find the radius (r) (by Case I. Prop. XIX) and the arc (a) in degrees (by the note to Prop. XX.) multiply the square of the radius, the arc in degrees, and the number 0,0087267 continually; call the product A.

From the square of the radius, take the square of half the chord; multiply the square root of the remainder by half the chord; call the product B.

Then B taken from A, will leave the area of the Segment.

Or $\overline{rr \times a} \times \frac{3{,}1416}{360} - \frac{1}{2}b\sqrt{rr - bb} = $ Area.

EX. *Suppose the chord of the arc of a circular segment is 48, and the height 18; what is its area?*

Now $\left(\frac{24 \times 24 + 18 \times 18}{2 \times 18} = \right)$ 25 is the radius.
And 147,483 are the degrees in the arc.
Then $(25 \times 25 \times 147{,}483 \times 0{,}0087267 =)$ $804{,}4015 = A$.
And $(\frac{48}{2} \times \sqrt{25 \times 25 - \frac{48}{2} \times \frac{48}{2}} =)$ $168 = B$.
Therefore $(A - B =)$ 636,4015 is the area of the segment required.

PRO-

133. PROPOSITION XXIII.

In a circular zone (BDdb) or that part of a circle contain'd between two parallel chords, (BD = 2B, bd = 2b) the length of those chords, and their distance (Aa = h being known. Fig. 32.

I. *To find the distance* (CA = *x*) *of the centre* (C) *of that circle, from the middle* (A) *of the greater chord.*

RULE. To the square of the distance of the chords, add the square of half the lesser chord.

The difference between this sum and the square of half the greater chord, divided by twice the distance of the chords, will give the distance of the centre as required.

$$\text{Or } x = \frac{\pm BB \mp bb + hh}{2h}$$

Note, If the said sum

$$\text{is } \left\{\begin{array}{l}\left.\begin{array}{l}\text{greater}\\ \text{less}\end{array}\right\}\text{than}\\ \text{equal to}\end{array}\right\}\text{ the square of the half chord.}$$

$$\text{the centre falls}\left\{\begin{array}{l}\left.\begin{array}{l}\text{between}\\ \text{without}\end{array}\right\}\text{ the two chords.}\\ \text{in the middle of the greater chord.}\end{array}\right.$$

EX. *Suppose the greater chord is* 48, *the lesser* 30; *and their distance* 13: *How far is the center of that circle distant from the middle of the greater chord?*

Now

Now $13 \times 13 + 15 \times 15 = 394$.
And $24 \times 24 = 576$.

Then $\left(\frac{576 - 304}{13 \times 2} =\right)$ 7 the diſtance of the centre as required.

Here the two chords are on the ſame ſide from the centre.

Note, The diſtance of the greater chord from the centre of the circle being known; the radius of that circle may be thus found.

RULE. To the ſquare of half the greater chord, add the ſquare of its diſtance from the centre; the ſquare root of the ſum, will be the radius required.

EX. *Thus, in the foregoing example, where half the greater chord is* 24; *and its diſtance from the centre is found to be* 7.

Then $(\sqrt{24 \times 24 + 7 \times 7} =)$ 25 is the radius of the circle.

II. *To find the height* (aE = v) *of the circular ſegment* (bEd) *whoſe baſe* (bd) *is the leſſer chord*.

RULE. To the ſquare of half the greater chord, add the ſquare of the breadth of the zone; from the ſum take the ſquare of half the leſſer chord; divide the remainder by the breadth; call the quotient, A.

To the ſquare of half the leſſer chord; add the ſquare of $\frac{1}{2}$ A; from the ſquare root of the ſum, take $\frac{1}{2}$ A; and the remainder is the verſed ſine, or height of the ſegment required.

Or

$$\text{Or } A = \frac{BB + bb - bb.}{b}$$

$$\text{Then } v = \sqrt{bb + \overline{\tfrac{1}{2}A}|^2} - \tfrac{1}{2}A.$$

Ex. *Suppose the greater chord is 48, the lesser 30; and their distance is 13: What is the height of the segment whose base is the lesser chord?*

$$\text{Now } \left(\frac{\overline{24 \times 24 + 13 \times 13 - 15 \times 15}}{13} = \right) 40 = A;$$

its half $= 20$.

And $15 \times 15 + 20 \times 20 = 625$; whose square root is 25.

Then $(25 - 20 =)\ 5$ is the height of the segment standing on the lesser chord.

And $(13 + 5 =)\ 18$ is the height of the segment whose base is the greater chord.

Note, The height of either of the segments, standing on the greater or lesser chords being known, the radius of the circle may be found by Case I. Prop. XIX.

134. PROPOSITION XXIV.

To find the area of a circular zone; its breadth and the length of its ends being known.

RULE I. Find the heights of (AE, *a*E) the circular segments (BED, bEd) on each end (BD, bd) by Case II. Prop. XXIII.

Find the diameter (by Case I. Prop. XIX.)

Then

Then the difference of the segments on the greater and lesser ends of the zone (found by Rule III. Prop. XXII.) will be the area of the zone.

Ex. *What is the area of a circular zone; one end being* 48, *the other end* 30; *and the breadth* 13?

By Case II. Prop. XXIII. the height of the segment on the lesser end of the zone is equal to 5.

And $13+5=18$ is the height of the segment on the greater end.

By Case I. Prop. XIX. the diameter is equal to 50.

Then 636,6114 is the area of the segment, on the greater end of the zone by Rule III. Prop. XXII.

And 102,1865 is the area of the segment, on the lesser end by the same.

Consequently 534,4249 is the area of the zone.

Rule II. 1. Find the distance of the centre of the circle from the greater end of the zone (by Case I. Prop. XXIII.)

2. Find the radius, (by the note to the same.)

3. Find the chord of the arc, between the ends of the zone; (Case I. Prop. IV.) and the versed sine of half the arc. (by Prop. XVII.)

4. Find the length of the arc by Case II. Prop. XX.)

5. Multiply half the lesser end of the zone, by the breadth thereof, call the product A.

6. Multiply the radius by the length of the arc, call the product B.

7. From

7. $\left.\begin{matrix}\text{From}\\ \text{To}\end{matrix}\right\}$ the sum of A and B $\left\{\begin{matrix}\text{take}\\ \text{add}\end{matrix}\right\}$ the product of the distance from the centre, by the $\left\{\begin{matrix}\text{diff.}\\ \text{sum}\end{matrix}\right\}$ of half the ends of the zone; and the $\left\{\begin{matrix}\text{remainder}\\ \text{sum}\end{matrix}\right\}$ will be the area of the zone; when the centre falls $\left\{\begin{matrix}\text{without}\\ \text{between}\end{matrix}\right\}$ the ends.

EX. *In a circular zone, suppose the greater end is 48, the lesser end 30, and the breadth 13: What is the area?*

Now $\left(\frac{24\times24-\overline{15\times15}+\overline{13\times13}}{2\times13}=\right)$ 7 is the distance of the greater chord from the center (which falls without the zone.)

And $(\sqrt{24\times24+7\times7}=)$ 25 is the radius.

Also $24-15=9$ is the difference between the half ends.

Then $(\sqrt{9\times9+13\times13}=)15{,}8114$ is the chord of the arc between the ends of the zone.

And $\left(25-\sqrt{25\times25-\frac{15{,}8114}{2}\times\frac{15{,}8114}{2}}=\right)$ 1,283 is the versed sine of half that arc.

Then $\left(1+\frac{\frac{2}{3}\times1{,}283}{50-0{,}82\times1{,}283}\times15{,}8114=\right)16{,}0875$ is the length of the arc between the two ends of the zone.

Now $(\frac{30}{2}\times13=)$ $195=A$.

And $(16{,}0875\times25=)$ $402{,}187=B$.

And $(402{,}187+195=)$ $597{,}187$ is the sum of A and B.

Then $(597{,}187-9\times7=)534{,}187$ is the area of the zone.

RULE.

RULE III. Find the area of one of the two circular segments contain'd in the given zone. (by Case II. Prop. XXII.)

Multiply half the sum of the ends of the zone, by the breadth; to the product add twice the area of the segment, before found; and the sum will be the area of the zone.

Ex. *In a circular zone; suppose the greater end is 48, the lesser end 30; their distance, 13; the base of one of the contain'd circular segments is 15,8114, and its height 1,283: What is the area of that zone?*

Now $1,283 \times 0,626 = 0,803158$.

And $0,803158 \times 0,803158 + \frac{15,8114}{2} \times \frac{15,8114}{2} = 63,145062$; whose square root is 7,9464.

And $\left(7,9464 \times 2 \times \frac{2 \times 1,283}{3} =\right) 13,5936$ the area of one segment.

Then $\left(\frac{48+30}{2} \times 13 + 13,5936 \times 2 =\right) 534,1872$ is the area of the zone.

135. PROPOSITION XXV.

To find the diameter of a circle, whose area shall be in a given proportion to that of a circle whose diameter is known.

RULE.

The given diameter $\left\{\begin{array}{c}\text{multiply'd}\\ \text{divided}\end{array}\right\}$ by the square root of the intended $\left\{\begin{array}{c}\text{increase,}\\ \text{decrease,}\end{array}\right\}$ will give the diameter of the circle required.

Ex. I. *What is the diameter of a circle, whose area is 9 times as much as one of 21 inches diameter?*

Then ($21 \times \sqrt{9} =$) 63 inches is the diameter of a circle 9 times as large as one 21 inches in diameter.

Ex. II. *What is the diameter of a circle, whose area is but $\frac{1}{9}$ of a circle of 21 inches diameter?*

Then $\left(\frac{21}{\sqrt{9}} =\right)$ 7 inches, is the diameter of the circle required.

136. PROPOSITION XXVI.

To find the area of figures, whose sides are partly right lines, and partly arcs of a circle.

RULE.

To every curved side draw a chord; and the given figure will be reduced to a right-lined one, and as many segments as there were curved sides. Then the sum of the areas of these parts, will be the superficial content of the given figure.

EXAMPLE.

Suppose a field of five sides, two whereof are the arches of circles; now the chords being drawn, the figure will be a five-sided right-lined figure, which being (by lines drawn) divided into a trapezium and a triangle; in the trapezium, the diagonal is 7 chains, and the sum of the perpendiculars $6\frac{1}{2}$ *chains; in the triangle, the base is 4 chains, and the perpendicular* 6 *chains; in one segment, the whole chord is* 6 *chains, and the chord of half the arch is* $3\frac{1}{2}$ *chains; in the other segment, the whole chord is* 4 *chains, and the chord of half the arch is* $2\frac{1}{2}$ *chains; what is the area of this figure?* Fig. 37.

First,

First, $\frac{7}{2}=3,5$ then $3,5\times 6,5=22,75$, the area of the trapezium;

Secondly, $\frac{6}{2}=3$, then $3\times 4=12$, the area of the triangle;

Thirdly, In one ſegment the chord of half the arch is 3,5, and half the whole chord is 3, then $3,5\times 3,5=12,25$; and $3\times 3=9$; and $12,25-9=3,25$, whoſe ſquare root is 1,8 the verſed ſine.

And $\frac{6}{2}\times\frac{6}{2}+\frac{1,8\times 1,8}{4}=9,81$; whoſe ſquare root is 3,132.

Then $\frac{3,132\times 8+3,5\times 2}{15}\times 1,8\times 2=7,6932$ the area *per* Rule II. Prop. XXII.

Fourthly, In the ſecond ſegment, the chord of half the arc is 2,5; and half the whole chord is 2.

Then $\sqrt{2,5\times 2,5-2\times 2}=1,5$ the verſed ſine.

And $\frac{4}{2}\times\frac{4}{2}+\frac{1,5\times 1,5}{4}=4,5625$; whoſe ſquare root is 2,136.

Then $\frac{2,136\times 8+2,5\times 2}{15}\times 1,5\times 2=4,416$ the area (by Rule II. Prop. XXII.)

Now

Now $22,75 + 12 + 7,693 + 4,416 = 46,859$ square chains for the area of the field.

And as 10 square chains make 1 acre.

Therefore $\frac{46,859}{10} = 4$ acres 2 roods 33 poles.

SECTION VII.

437. PRACTICAL QUESTIONS.

QUESTION I.

A Round pillar 7 inches over, is sufficient to carry a certain weight; of what diameter is the column that contains 10 *times the stone on the same length?*

Now $7 \times 7 \times 10 = 490$, whose square root is 22,135 inches, the diameter required.

QUESTION II.

A brewer has a cistern which is fill'd by three pipes, each of 3 *inches bore; of what diameter must the bore of that pipe be, which in the same time, will throw him in* $2\frac{1}{2}$ *times as much water?*

The quantity of water thrown in, being as the squares of the diameters;

Therefore $3 \times 3 \times 3 \times 2,5 = 67,5$, whose square root is 8,215 inches, &c. the diameter sought.

QUESTION III.

If a piece of a cable, 3 feet long, and 9 inches in compass, weighs 22 lb; what will a fathom weigh, of that cable, whose diameter is 9 inches?

Now 1 *circumf.* : 0,31831 *diam.* : : 9 *circumf.* : 2,86479 *diam.* whose square is 8,2069, &c.

Then 8,2069 : 22 lb : : 81 (9 × 9) : 217,14, which × by 2, gives 434,28, the weight sought.

QUESTION IV.

I want in a garden a circular pond, that shall just take up half an acre; how long must the cord be that will strike the circle?

The half acre contains 2420 square yards;

Therefore 1 : 1,2723 : : 2420 : 3081,1441 the square of the diameter, whose square root is 55,508 the diameter; the half of which, 27,75 yards, is the length of the line sought.

QUESTION V.

A carpenter is to put an oaken curb to a round well, at 8 d. per foot square; the breadth of the curb is to be $7\frac{1}{2}$ inches, and the diameter within is $3\frac{1}{2}$ feet, what will be the expence?

Now $3\frac{1}{2}$ *f.* = 42 *in.*

Then 42 × 42 × 0,7854 = 1385,4456, the area within the curb.

Also 42 + 7,25 + 7,25 = 56,5, the outside diameter of the curb.

And $56,5 \times 56,5 \times ,7854 = 2507,19315$ the area of a circle including the well and curb.

Then $2507,19315 - 1385,4456 = 1121,74755$, the area of the curb.

Now 8*d*. = 0,03 £.

Ther. as 144 : 0,03 : : 1121,74755 : 0,259671 £. = 5*s*. $2\frac{1}{4}$*d*. the expence sought.

Or. The diameters of two circles being known, the difference of their areas may be found by the following

RULE. Multiply the sum of the diameters, the difference of the diameters, and 0,7854 continually; the product will be the difference of the areas.

Thus $\overline{56,5+42} \times \overline{56,5-42} \times 0,7854 = 1121,74755$ is the area of the curb.

QUESTION VI.

There is wanted in a garden a circular pond with a circular island in the middle; the diameter of the pond must be 100 *yards, and the circumference of the island the same; what will the digging of the pond come to at* 18 *d. per square yard on the surface?*

Now $100 \times 100 \times 0,7854 = 7854$ yards, the area of the pond and island.

And $100 \times 100 \times 0,07958 = 795,8$, the area of the island.

Then $7854 - 795,8 = 7058,2$ yards the area of the pond.

And 1 : ,075 £. : : 7058,2 : 529,365 £ = 529 £. 7 s. $3\frac{1}{4}$ d.

QUES-

QUESTION VII.

Suppose the expence of paving a semicircular plot, at 2 *s.* 4 *d. per foot, amounted to* 10 *£. what is the diameter thereof?*

Now 2 *s.* 4 *d.* = 0,11$\dot{6}$ *£.*

Therefore 0,11$\dot{6}$ *£.* : 1 *f.* : : 10 *£.* : 85,714$\frac{2}{7}$ the semicircle's area.

And 85,714$\frac{2}{7}$ × 2 = 171,428$\frac{4}{7}$, the circle's area.

Then 1 : 1,2732 : : 171,428$\frac{4}{7}$: 218,26285713, whose square root is 14,7737 the diameter sought.

QUESTION VIII.

Suppose St. James's *square to be* 180 *yards long, and* 150 *yards broad, in which there is a regular octagonal gravel walk, one of whose sides is (suppose)* 28 *yards; what did the paving the rest with Purbeck stone come to, at* 3 *s.* 6 *d. per yard?*

Now 180 × 150 = 27000 yards, the area of the square.

And 28 × 28 + 4,828427 = 3785,486768 yards the area of the octagon.

Then 27000 yards — 3785,486768 = 23214,513232 yards, what was paved.

And 1 *yard* : 0,175 *£.* : : 23214,513232 yards : 4062,5397; = *£.* 4062. 10 *s.* 9$\frac{1}{2}$ *d.*

QUESTION IX.

What is the area of the segment of a circle whose diameter is 50 *inches; supposing the section made* 14 *inches from the centre?*

I 4 Now

Now $\frac{50}{2} = 25$, the radius, and $25 - 14 = 11$; the versed sine or height.

Then by Prop. XXII.

$$\frac{11 \times 32}{80 \times 50 - 15 \times 11} = 0{,}091786$$

And $\frac{4}{3} - 0{,}091786 = 1{,}241547$

And $\sqrt{50 \times 11} = 23{,}452$.

Then $(1{,}241547 \times 23{,}452 \times 11) = 320{,}28456$ is the area of the segment.

QUESTION X.

A, B, and C, bought a circular cheese, 14 inches in diameter, which cost them 7 s. 6 d. whereof A pays 1 s. 4 d. B 2 s. 10 d. and C 3 s. 4 d. now they agree that it shall be divided from the centre to the circumference; that is, it should be cut into three sectors; whose areas should bear the same proportion to each other, as the prices paid; what part of the circumference will fall to each man's share, together with the areas.

Now $14 \times 14 = 196$, and $1 : ,7854 :: 196 : 153{,}9384$, the whole area.

Also 7 *s.* 6 *d.* = ,375 £. 1 *s.* 4 *d.* = ,06̇ £. 2 *s.* 10 *d.* = ,141̇6 £. 3 *s.* 4 *d.* = ,16̇ £.

And $\frac{153{,}9384}{0{,}375} = 410{,}5$.

$$\text{Then } \left.\begin{array}{l} 410{,}5 \times 0{,}0\dot{6} = 27{,}3\dot{6} = \text{A's} \\ 410{,}5 \times 0{,}141\dot{6} = 58{,}1541\dot{6} = \text{B's} \\ 410{,}5 \times 0{,}1\dot{6} = 68{,}41\dot{6} = \text{C's} \end{array}\right\} \text{share of the area.}$$

Again, $1 : 3{,}1416 :: 14 : 43{,}9824$ = circumf.

And

And $\frac{43,9824}{0,375} = 117,286$.

Then $117,286 \times 0,06 \;\; = 7,819 = A$'s share
$117,286 \times 0,1416 = 16,615 = B$'s of the
$117,286 \times 0,16 \;\; = 19,547 = C$'s circu.

QUESTION XI.

A, B, C, bought a grinding stone of 21 inches in diameter; each paying a third; what part of the diameter must each grind down?

This question is answer'd, by reckoning each man to grind away one third of the circular area.

Now $21 \times 21 \times ,7854 = 346,3614$, the area.

Then $\frac{346,3614}{3} = 115,4538$, each man's area.

which taken from the whole, leaves 2 0,9076, for the area of the circle remaining, when one man has ground away his share.

And 1 : 1,27324 : : 230,9076 : 294, whose square root is 17,14, the diameter of two men's shares next the centre.

Again, 1 : 1,2732 : : 115,4538 : 147, whose square root is 12,12, the diameter of one man's share next the centre.

Then 21—17, 12=3,86 is the breadth of the ring for the 1st man's share.

Also 17,14—12,12=5 is the breadth of the ring which the 2d man is to grind away.

And 12,12 is the diameter of the 3d man's share.

QUESTION XII.

A workman is employed to set up a rail round a circular bason at 12 feet distance; and to lay a gravel walk between the rail and bason: The price of the rail at 5 s. a linear yard, and the walk at 1 s. 6 d. a square yard: Now a line of 28,1267 yards stretch'd close by the brink of the bason, will with both ends touch the rail: What will the workman's bill amount to?

I 5 Now

Now $\overline{\frac{28,1267}{2}}\Big|^2 = 197,1$.

Then $\frac{197,1}{4} + 4 = 53,4$ the diameter of the rail.

And $1 : 3,1416 :: 53,4 : 167,90106$ or $167,9$, the circumference or length of the rail.

Therefore $\frac{53,4}{2} \times \frac{167,9}{2} = 2243,3301$ the area, including the walk and bason.

Now 12 f. = 4 yards.

And $\overline{53,4 - 4 \times 2} = 45,4$ the diameter of the bason.

Then $\overline{53,4 + 45,4} \times \overline{53,4 - 45,4} \times 0,7854 = 621,33342$ square yards is the area of the gravel walk.

Now $167,9 \times 0,25$ £. $= 41,975$ £. the price of the rail.

And $621,33342 \times 0,075$ £. $= 46,6$ the price of the walk.

Therefore 88 £. 11 *s.* 6 *d.* is the whole expence.

Question XIII.

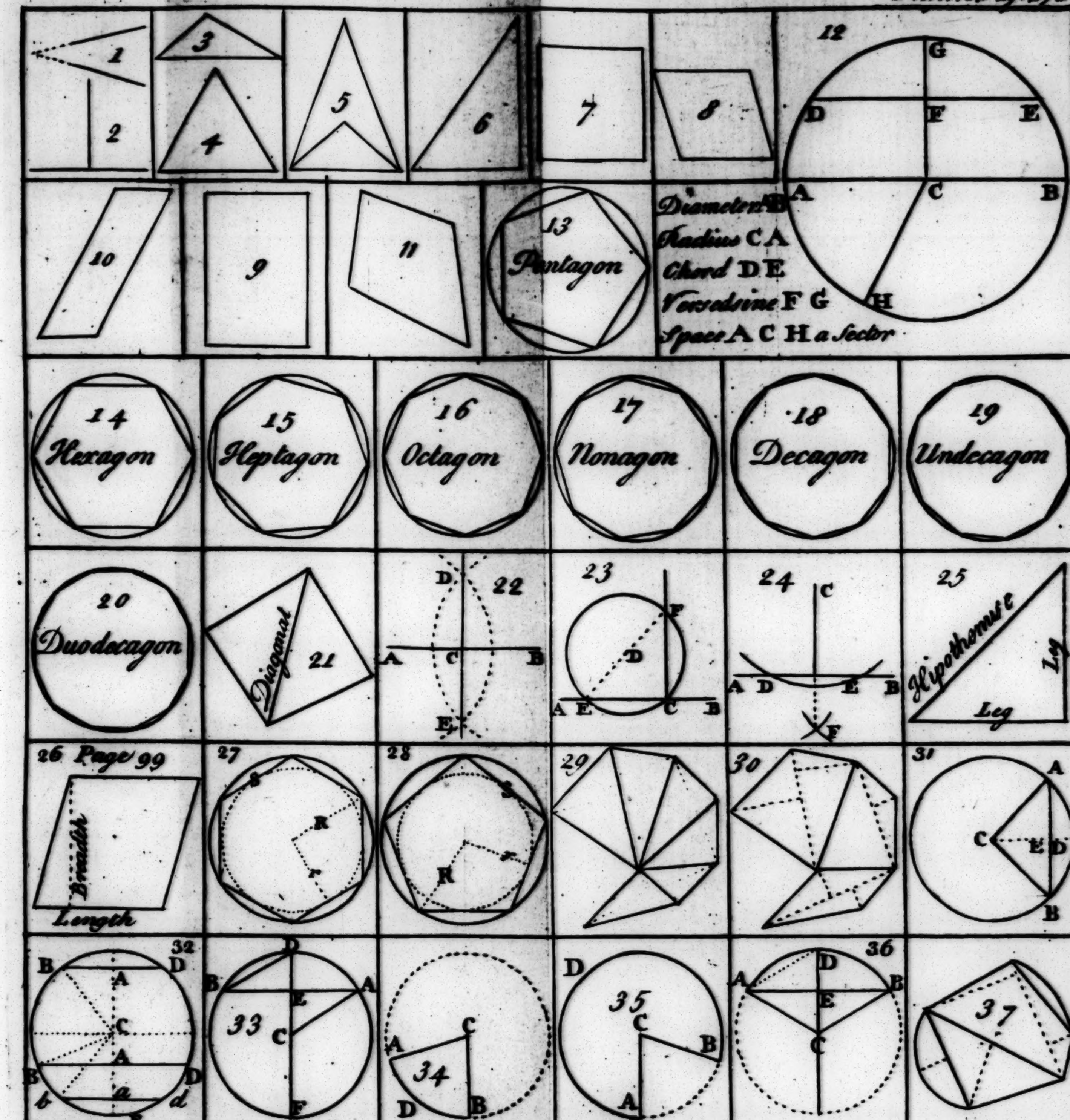
A gentleman has ordered a square plat of two acres to be laid out fronting his house; in this plat he would have a regular octagonal bason, containing a quarter of an acre, and four of its sides parallel to the sides of the plat; also gravel walks of 8 yards wide running diagonally, and from the middle of each side, terminating in a walk of the same breadth, surrounding the bason: What will the expence of gravelling these walks come to, at 16 d. a square yard?

Now 4840 yards = 1 acre.

Therefore $\sqrt{4840 \times 2} = 98,3869886$ is the side of the plat.

And

Plate. I. Page 178



And $\frac{\sqrt{4840}}{4} = 34,7850543$ a side of a square equal to the octagonal bason.

Then $34,7850543 \times 0,549342 = 19,1088902$ is the radius of the inscrib'd circle.

Therefore $\frac{98,3869886}{2} - 19,1088902 - 8 =$ 22,0846041 is the length of one of the walks from the sides of the plat.

Then $22,0846071 \times 8 \times 4 = 706,7074592$ is the area of those four walks.

Again, $\overline{\sqrt{4840} + 2 \times 2} = 139,140217$ is the diagonal of the square plat.

And $\frac{139,140217}{2} - 19,1088902 - 8 = 42,$ 461268 is the length (in the middle) of one of the diagonal walks.

Then $42,461268 \times 8 \times 4 - 64 = 1294,$ 760576 is the area of the four walks in the diagonals.

Now $\overline{19,1088902 + 8}|^2 \times 3,3137084 = 2435,$ 2175541 is the area of the octagonal walk and bason.

And $2435,2175541 - 1210 = 1225,2175541$ is the area of that walk.

Then 3226,685589 is the area of all the gravel walks.

Hence the expence will be 215 £. 2 *s.* 3 *d.*

PART II.

Of solid MEASUREMENT.

Wherein will be shewn,

First, *Definitions of the more common solids.*

Secondly, *Methods of measuring, or of cubing of timber.*

Thirdly, *Methods of calculating the solidities and superficies of various sorts of solid figures.*

SECTION I.

DEFINITIONS.

138. A Solid is a figure contained under three dimensions, *viz.* length, breadth, and depth, or thickness.

139. A solid whose bases or ends are equal, parallel and like rectiline plane figures, and whose sides are parallelograms, is called a *prism*; and is denominated from the number of the sides of its base.

140. If

140. If the ends and sides are equal squares, the solid is call'd a *cube*. *Plate* 2. *Fig.* 1.

141. If the base or end is a rectangle; the solid is call'd a *parallelopiped*. *Fig.* 2.

142. * If the base or ends are circles, the solid is called a *cylinder*. *Fig.* 3.

143. A solid formed on a plane rectiline base, having on its sides right-lined plane triangles, whose vertices all meet in one and the same point; is call'd a *pyramid*; and is denominated from the number of the sides of its base. *Fig.* 4.

144. * If the base is a circle, the solid is call'd a cone. *Fig.* 5.

145. That solid which is terminated by a convex surface, whereof every point is equally distant from a certain point within the solid, is call'd a *sphere*.——The right line passing thro' that point, equally distant from the surface, is called the *diameter* or *axis*. *Fig.* 6.

146. In a pyramid, cone, sphere, or any other tapering solid, a part thereof contain'd between two parallel ends, is called a *frustum*: And the parts wanting, at the ends of a *frustum*, to compleat the tapering solid, are called *segments*.

147. If a frustum of a tapering solid be cut by a plane diagonally, from the extremity of one side at the lesser end, to the extremity of the opposite side at the other end, each of these pieces is called a *hoof*, or an *ungula*; that being the greatest which has the greatest base.

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* *Note*, In Def. 142 and 144, the number of sides in the base or end, is supposed to be exceeding great, or infinite, and such figures may be taken for curves.

SEC.

SECTION II.

148. *Of the measuring of timber trees.*

WHEN trees are to be measured, their length is denoted by feet and inches; or by feet and tenth parts: And it has been usual to take their circumference by a small cord, or chalk line; and this measure is denoted by the denominations that the length was taken in.

Workmen and measurers have usually divided their compass or circumference by 4; and this $\frac{1}{4}$ part, they call the *girt*, and estimate it as though it was the side of a square, whose area was equal to a section of the tree, at the place it was girted; but in the Royal Dock yards this method is only used for ashe unstript of its bark.

It has been a customary allowance to the buyer, in trees of an uniform growth, to take the girt any where he pleases, between the greater end and the middle of the tree.

Oak, elm and beech is squared by the merchant before it is served into the King's yards; and by their contract the sum of the breadth of the *slabs* taken off, are not to be less than twice the sum of the *wanes*; if they should be less, the King's measurers hew the opposite sides, until the dimensions are reduced to the terms of the contract; generally giving the turn of advantage to the merchant.

In the King's yards, the sides of timber thus squared, are by a pair of callipers (or large compasses whose legs are so bent, that the points will meet) taken between the points; and these extents being measured on a scale of inches, give the dimensions to be used instead of the said *girt*.

There

There are several other articles that enter into the measuring of timber for sale, but they will be best understood by treating of them as particular cases of the following proposition.

149. PROPOSITION I.

To compute the solidity of round timber, by having the length and circumference given?

CASE I. *When the tree is strait, and the ends are equal; or nearly so.*

RULE. Multiply the square of one fourth of the circumference, by the length, and the product is the solidity, or (as commonly call'd) the content.

EXAMPLES.

I. *What is the solid content of a tree, whose compass is 32 inches, and the length 9 feet?*

Now $\frac{32}{4} = 8$ inches.

By decimals.

8 inches = 0,6̸ feet
mult. by 0,6̸
9)40 (see art. 97.)
,44
× 9 the length.
4,00 = 4 solid feet.

By duodecimals.

f. i.
0 . 8 } mult.
0 . 8 } mult.
0 . 5 . 4
× 9
4 . 0 . 0

II. *What*

II. *What is the solidity of a tree, whose length is 31 feet, and the ¼ of the compass is 16 inches?*

By decimals.

16 inc. = 1,3 feet
× 1,3
44 (see art. 99.)
133
177
× 31 the length
177
5333
55,11 feet the solidity.

By duodecimals.

f. i.
16 inc. = 1 . 4 } ×
1 . 4
0 . 5 . 4
1 . 4 .
1 . 9 . 4
31
0 . 10 . 4
23 . 3
31
55 . 1 . 4

III. *What is the solidity of a tree, whose ¼ compass is 11 inches; and the length 40 ½ feet?*

By decimals.

11 inc. = 0,916 feet
× 0,916
9)5500
6111
9166
8250
,84027
× 40,5 the length
420138
336111111
34, 3125 feet, the solidity.

By duodecimals.

f. i.
0 . 11 } ×
0 . 11
0 . 10 . 1.
× 40 . 6 . = length.
0 . 5 . 0 . 6
0 . 3 . 4
33 . 4
34 . 0 . 4 . 6

IV. *Suppose a piece of timber is 9 $\frac{3}{4}$ feet long; and $\frac{1}{4}$ of the circumference is 39 inches: What is the solidity?*

By decimals.

```
39 inch. = 3,25 feet
         × 3,25
           1625
           650
          975
        10,5625
length =  57,9  inverted
          95062
           7394
            528
        102,984 feet, the solidity.
```

By duodecimals.

```
                f.  i.
39 inch. =      3 . 3
              × 3 . 3
                0 . 9 . 9
                9 . 9
               10 . 6 . 9
         × by   9 . 9 = len.
                7 . 11 . 0 . 9
               95 . 0 . 9
              102 . 11 . 9 . 9
```

V. *A tree whose $\frac{1}{4}$ of the compass is 31 inches, and the length 24 feet: What is the solidity?*

By decimals.

```
31 inc. = 2,583 feet
        × 385,2 inverted.
          5167
          1292
           206
             8
         6,673
   ×        24  the length
         26692
         13346
       160,152 feet the solidity.
```

By duodecimals.

```
              f.  i.
31 inc. =     2 . 7
          ×   2 . 7
              1 . 6 . 1
              5 . 2
              6 . 8 . 1
          ×  24
              0 . 2 . 0
             16 . 0
            144
            160 . 2 . 0
```

Note, If the tree is crooked, its length must not be measured on either the convex, or concave side of the curve.

150. CASE II.

150. CASE II. *When the tree is taper, or unequally thick.*

RULE. Gird the tree in as many places as are thought neceſſary: the ſum of the ſeveral girds divided by their number, gives (as thought by workmen,) the mean compaſs; and the fourth of the mean compaſs ſquared and multiplied by the length, gives the ſolidity.

Ex. VI. *A tapering tree* 15 *feet long, is girded in four places; in the firſt, it is* 5 *f.* 9 *in.; the ſecond,* 4 *f.* 5 *in.; the third* 4 *f.* 9 *in.; and the fourth, is* 3 *f.* 9 *in.; what is the ſolidity?*

	f. i.
The 1ſt compaſs	5 · 9
2d - - -	4 · 5
3d - - -	4 · 9
4th - - -	3 · 9
the number of girds = 4)18 . 8(	
	4 . 8 the mean compaſs.

f. i.

And $\frac{4 . 8}{4}$ = 1. 2. the quarter compaſs.

```
  f.  i.
  1 . 2
 ×1 . 2
 ---------
  0 . 2 . 4
  1 . 2
 ---------
  1 . 4 . 4
 ×15         = length.
 ---------
 20 . 5 . 0 the ſolidity.
```

151. Becauſe

151. Because of the great irregularity in the growth of timber, especially such as is most useful in ship-building. The taking of a mean out of several girts or dimensions not being sufficiently accurate, the method now chiefly used, is to measure the tree into as many uniform lengths as the measurers shall judge it proper; and then to multiply each length by its proper transverse dimensions; and by adding the solidities of the several lengths together, obtain the solidity of the whole.

152. Case III. Branches or boughs measuring two feet in compass; (or 6 inches girt) are reckon'd as timber, and their solidity is to be computed and added to that of the tree.

So much of the trunk, as measures less than 2 feet compass, is not esteemed timber.

Ex. VII. *Required the solidity of a tree 37 feet long; and 22 inches, quarter compass; one branch 12 feet long, by 28 inches circumf. and another, 8 feet long, by 24 inches compass?*

Tree	branch	branch
1 . 10	$\frac{28}{4}$ = 0 . 7	$\frac{24}{4}$ = 0 . 6
× 1 . 10	× 0 . 7	× 0 . 6
1 . 6 . 4	0 . 4 . 1	0 . 3
1 . 10	+ 12	+ 8
3 . 4 . 4	4 . 1	2 . 0
× 37		
1 . 0 . 4	Tree =	124 . 4 . 4
12 . 4	one branch =	4 . 1 . 0
111	other branch =	2 . 0 . 0
124 . 4 . 4	solidity =	130 . 5 . 4

153. CASE IV. *When the trees have their bark on.*

In measuring such timber for sale, 'tis common to make an allowance to the buyer, on account of the bark; thus in oak, $\frac{1}{10}$ or $\frac{1}{12}$ part of the circumference is deducted; but the allowance for the bark of elm, beach, ashe, &c. is less: This deduction being made, is supposed to reduce the compass, to that which the tree will have, when the bark is stripped off. Therefore,

RULE. From the given circumference, subduct the allowance for bark; and with the remaining compass find the solidity as before.

Ex. VIII. *An oak tree is* 45 *f.* 7 *inches long, and* 3 *f.* 8 *inches quarter compass; required the solid content; allowing* $\frac{1}{12}$ *for bark?*

f. i.
12) 3 . 8
3 . 8 = allowance
3 . 4 . 4 = reduced quarter compass
× 3 . 4 . 4
0 . 1 . 1 . 5 . 4
1 . 1 . 5 . 4
10 . 1 . 0
11 . 3 . 6 . 9 . 4
× 45 . 7 = length.
6 . 7 . 0 . 11 . 5 . 4
0 . 0 . 1 . 3 . 0
0 . 2 . 9 . 9
1 . 10 . 6
11 . 3
495
feet 514 . 11 . 5 . 11 . 5 . 4 the solidity.

154. CASE V. *When timber is to be reduced to loads.*

Measurers and workmen, reckon 40 feet of *unhewn*, or *rough timber*, and 50 feet of *hewn timber*, to

to a load, suppos'd to weigh a ton, or 20 hundred weight: For, say they, *hewn timber* is measured by the square, and is very near exact; but *rough timber*, by the *girt*, (or quarter compass,) which is about $\frac{1}{5}$ less than exact; therefore in the buying of timber, it amounts to much the same, whether it be measured by the *girt*, at 40 feet solid to a load; or measured exact, at 50 feet to a load; hence these

RULES: Divide the feet, in rough timber, by 40, gives the Loads.

Divide the feet, in hewn timber, by 50, gives the Loads.

In the King's yards, 40 feet of hewn timber is reckoned a ton; and 50 feet of such timber goes to a load.

Ex. IX. *How many loads of timber is in that rough tree; whose length is 28 feet six inches, and the quarter compass, 2 feet 9 inches.*

```
     f.  i.
     2 . 9
  ×  2 . 9
     2 . 0 . 9
     5 . 6
     7 . 6 . 9
  ×  28 . 6      the length
     3 . 9 . 4 . 6
     1 . 9 . 0
     14 . 0 .
     196
 40)215 . 6 . 4 . 6   the solid content.
     5 3/8 loads.
```

In the foregoing examples, are contained all the varieties that occur in the measuring of rough timber for sale: But when timber is regularly and smoothly hewn, the solidities of such pieces had best

beſt be computed by the rules given for *priſms*, *pyramids*, *cones*, &c. and their *fruſtums*; as will be ſhewn hereafter.

155. Although the foregoing method of computing the ſolidity by the fourth part of the circumference, is commonly uſed by artificers, on account of its eaſe, yet in fact, it is very erroneous, for the fourth part of the circumference, of a circle, cannot be equal to the ſide of a ſquare of equal area to that circle. Thus, if the circumference of a circle be 1, the ſide of a ſquare of equal area to that circle, is 0,2821; whereas by the falſe method of the *girt*, it is but 0,25.

Now the ſolidity of a round tree, may be found near enough, by either of the following rules.

I. Multiply the ſquare of the tree's compaſs by the length; and this product, by 0,07958, will give the ſolidity.

II. Multiply the ſquare of the tree's compaſs by $\frac{1}{12}$ of the length; divide this product by 24; to the quotient add a tenth of itſelf; this ſum, ſubtracted from the former product, leaves the ſolidity very near.

Note, inſtead of dividing by 24, it will be more convenient to divide by 6, and the quotient by 4.

III. Find the ſolidity by the common method of the girt; under this write its $\frac{1}{5}$ part; $\frac{1}{5}$ of this $\frac{1}{5}$ part; $\frac{1}{10}$ of this $\frac{1}{5}$ part; and the ſum of theſe four lines will be the ſolidity very nearly.

The two latter rules are beſt adapted for duodecimals, and the firſt for decimals.

The ſecond of the foregoing examples, is here wrought by each rule, both by decimals, and duodecimals.

The

The length 31 feet, and ¼ of the compass 16 inches; required the solidity.

First. By decimals.

The compass $= (16 \times 4 =)$ 64 inches $= 5,\dot{3}$ feet.

By rule 1st.

$$\begin{array}{r} 5,\dot{3} \\ \times\ \underline{5,\dot{3}} \\ 1\dot{7}\dot{7} \\ \underline{2\dot{6}66} \\ 28,\dot{4} \\ \times\ \underline{31} \end{array} = \text{length}$$

$$\begin{array}{r} 284 \\ \underline{8\dot{5}33} \\ 881,777 \\ \times\ \underline{85970,0} \\ 61724 \\ 7936 \\ 441 \\ \underline{70} \\ \underline{\underline{70,171}} \end{array}$$

By rule 2d.

$2,84\dot{4}$ = square of compass

$\frac{1}{12}$ feet $= \underline{2,58\dot{3}} = \frac{1}{12}$ of length

$$\begin{array}{r} 948\dot{1}\ \text{(see art. 99.)} \\ 2275\dot{8}5 \\ 142222\dot{2} \\ \underline{5688888} \\ 24 = 6 \times 4)\ 73,4814\dot{7} \\ -\ \underline{3,3678} \\ \underline{\underline{70,1136}} \end{array}$$

$(\underline{12,2469} = \frac{1}{6}$

$\left.\begin{array}{l} 3,0617 = \frac{1}{4} \text{ of } \frac{1}{6} \\ \underline{3061} = \frac{1}{10} \text{ of } \frac{1}{24} \end{array}\right\}$ add

$3,3078$ = sum

By rule 3d.

$55,11$ = solidity by the girt.

$11,022 = \frac{1}{5}$

$3,674 = \frac{1}{3}$ of $\frac{1}{5}$

$\underline{0,367} = \frac{1}{10}$ of $\frac{1}{3}$

$\underline{\underline{70,173}}$

Secondly,

Secondly. By duodecimals.

By rule 2d.

```
                   f.  i.
                   5 . 4 = compass.
                   5 . 4
                   1 . 9 . 4
                  26 . 8
                  28 . 5 . 4
31/12 feet =       2 . 7     = 1/12 length.
                  10 . 7 . 1 . 4
                  56 . 10 . 8
24=6×4)           73 . 5 . 9 . 4 (12 . 2 . 11 . 6 . 8 = 1/6
subtract           3 . 4 . 4 . 2   3 . 0 . 8 . 10 . 8 = 1/4 of 1/6
the solidity =    70 . 1 . 5 . 2   3 . 7 . 3 . 3 . 3 = 1/10 1/24
                                   3 . 4 . 4 . 1 . 11
```

By rule 3d.

```
 f.   '   "
55 . 1 . 4 by ex. 2d.
11 . 0 . 3 = 1/5
 3 . 8 . 1 = 1/3 of 1/5
 0 . 4 . 5 = 1/10 of 1/3
70 . 2 . 5 the solid content.
```

By each of these operations, the result is about 70 $\frac{1}{8}$ solid feet; but by the common method, the solidity is found to be only 55 $\frac{1}{10}$ feet; making a difference of above 15 feet, which is too considerable to be neglected.

Ex. XI. *Suppose the length of a tree, be 36 $\frac{1}{4}$ feet, and the compass 87 inches: What is the solid content?*

By decimals.

87 inches = 7,25 feet. Then by rule 1.
7,25 × 7,25 × 36,75 × 0,07958 = 153,722 the solidity.

By duodecimals.

	f. i.	
By rule 2d.	7 . 3	= 87 inches
	7 . 3	
	1 . 9 . 9	
	50 . 9	
	52 . 6 . 9	
$\frac{36\frac{1}{4}}{12}$ feet =	3 . 0 . 9	= $\frac{1}{12}$ of the length.
	3 . 3 . 5 . 0 . 9	
	157 . 8 . 3	
24=12×2)	160 . 11 . 8	(13 . 4 . 11 . 8 = $\frac{1}{12}$.
subtract	7 . 4 . 6	6 . 8 . 5 . 10 = $\frac{1}{2}$ of $\frac{1}{12}$.
the solid. =	153 . 7 . 2	8 . 0 . 7 = $\frac{1}{16}$ of $\frac{1}{24}$.
		7 . 4 . 6 . 5

K

By

By rule 3d.

	f	′	″		
$\frac{87}{4}$ = 21 $\frac{3}{4}$ inches =	1 .	9 .	9		
	1 .	9 .	9		
	1 .	4 .	3 .	9	
	1 .	4 .	3 .	9	
	1 .	9	9		
	3 .	3 .	5		
	×36 .	9			= length
	2 .	5 .	0 .	9	
	1 .	3 .	0		
	9 .	0			
	108				
The solidity	120 .	8 .	6 .	9	by the girt.
the correction	24 .	1 .	8 .	6	= $\frac{1}{5}$
	8 .	0 .	6 .	10	= $\frac{1}{3}$ of $\frac{1}{5}$
		9 .	7 .	3	× $\frac{1}{10}$ of $\frac{1}{3}$
the solidity	153 .	8 .	5 .	4	

It has been already observ'd, that in multiplying feet, inches, and parts, by feet, inches, and parts, the superficies express'd by the product, may consist of five denominations; among which, are two names beside square feet, square inches, and square parts: So in cubing of linear dimensions, the solid expressed by the product, may have seven different places; among these are four terms, beside cubic feet, cubic inches, and cubic parts; that is, two places between cubic feet, and cubic inches; and two places between cubic inches, and cubic parts.

To reduce, to cubic inches, the names between its place, and that of feet.

RULE.

RULE. Multiply the left-hand place by 12; to the product add its right-hand place; and 12 times this sum will be cubic inches.

The same rule will serve to reduce to cubic parts, the names between its place, and that of cubic inches.

Ex. *Suppose the solidity of a body was expressed by* 10f. . 8′ . 7″ . 9‴ . 5IV . 11V . 4VI *required the equivalent cubic feet, inches and parts?*

Here the place of thirds, is cubic inches; and that of sixths, is cubic parts.

Therefore 8′ . 7″ . 9‴ . = 103″ . 9‴ = 1245‴.

And 5IV . 11V . 4VI = 71V . 14VI = 866VI.

Consequently 10c.f. . 1245c.i. . 866c.p. = 10f. . 8′ . 7″ . 9‴ . 5IV . 11V . 4VI.

Here follows an explanation of the several names arising from the cubing of the linear dimensions of solids.

1. The feet, are cubic feet.

2. The primes, are parallelopipeds, of a foot long, a foot wide, and an inch thick.

3. The seconds, are parallelopipeds, either of a foot long, a foot broad, and a part thick. Or, of a foot long, an inch broad, and an inch thick.

4. The thirds are, either, parallelopipeds of a foot long, an inch wide, and a part thick; or, are cubic inches.

5. The fourths, are parallelopipeds; of a foot long, a part broad, and a part thick; or, of an inch long, an inch broad, and a part thick.

6. The fifths, are parallelopipeds, of an inch long, a part wide, and a part thick.

7. The sixths, are cubic parts.

SECTION III.

Of divers solids, bounded by right-lin'd *and* circular figures.

PROPOSITION II.

Given the linear dimensions of a cube, parallelopiped, cylinder, or of any prism. To find the solidity.

RULE.

Multiply the area of the base, or end, by the length, or height, and the product will give the solidity.

EXAMPLE I.

What is the solidity of a cube, whose side is 12 *inches?*

Now $12 \times 12 = 144$, the area of the end.
And $144 \times 12 = 1728$, the inches in a foot.

EXAMPLE II.

What is the solidity of a block of marble, whose length (AB) *is* 10 *feet, breadth* (AC) $5\frac{1}{2}$ *feet, and depth* (AD) $3\frac{1}{2}$ *feet?* Pl. 2. Fig. 2.

Now

Now 5,75 × 3,5 = 20,125, the area of the end.

And 20,125 × 10 = 201,25 feet, the solidity.

EXAMPLE III.

What is the solidity of a triangular prism, whose length is 18 feet, and one side of the quilateral end is 1½ feet?

The area of an equilateral triangle, whose side is 1, is 0,433013. (by tab. I. p. 144.)

Therefore, 1,5 × 1,5 × 0,433013 = 0,97427925, the area of the end.

Then, 0,97427925 × 18 = 17,5370265, the solidity.

EXAMPLE IV.

What is the solidity of a cylinder, whose length (AB) *is five feet, and the diameter* (AC) *of the end is 2 feet?* Fig. 3.

Now 2 × 2 × 0,7854 = 3,1416, the area of the end.

And 3,1416 × 5 = 15,708 feet, the solidity.

PROPOSITION III.

To find the solidity of a right pyramid, the measure of its base, or greater end; and the perpendicular height, or distance of the ends being given.

RULE.

Multiply the area of the base, or end, by a third part of the altitude, or length; and the product is the solidity.

EXAMPLE I.

What is the solidity of a pyramid, whose height (AC) *is* 24 *feet; and the side* (BD) *of its square base is* 3 *feet?* Pl. II. Fig. 4.

Now $3 \times 3 = 9$, the area of the base.

And, $\frac{24}{3} = 8 = \frac{1}{3}$ of the height.

Then, $9 \times 8 = 72$, the solidity sought.

EXAMPLE II.

What is the solidity of a pyramid, whose height is 15 *feet, and one side of its hexagonal base is* 18 *inches?*

The area of a hexagon, whose side is 1, is found to be 2,598076. (by tab. I. p. 144.)

And $1{,}5 \times 1{,}5 \times 2{,}598076 = 5{,}845671$, the area of the base.

Then, $5{,}845671 \times \frac{15}{3} = 29{,}228355$, the solidity.

PRO-

PROPOSITION IV.

To find the solidity of a right cone, the length, or perpendicular height, and the diameter, or circumference of the base being given.

RULE.

Mult. the sq. of the $\left\{\begin{array}{l}\text{dia. by } 0{,}2618 \quad = \dfrac{p}{12}\\[2ex] \text{circ. by } 0{,}026526 = \frac{1}{3} \text{ of } \dfrac{1}{4p}\end{array}\right\}$

and the product by the height, gives the solidity.

EXAMPLE I.

What is the solidity of a cone, the diameter (A =) *of whose base is* 18 *inches; and the altitude,* (CD) 15 *feet?* Pl. II. Fig. 5.

Now 18 inches = 1,5.
Then $1{,}5 \times 1{,}5 \times 0{,}2618 \times 15 = 8{,}83575$ feet the solid content.

EXAMPLE II.

If the circumference of the base of a cone be 40 *feet, and the height* 50 *feet: What is the solidity?*

Then $40 \times 40 \times 0{,}0265 \times 50 = 2120$ feet the solidity.

PROPOSITION V.

In a right cylinder, the length, and the diameter or circumference of the end being given; to find the convex superficies.

RULE.

Multiply the number 3,1416 by the diameter, then the product multiplied by the length, will give the convex surface.

Or, Multiply the circumference by the length, and the product will be the convex superficies.

EXAMPLE I.

What is the convex superficies of a right cylinder, whose diameter (AC) *is* 30 *inches; and the length* (AB) 60 *inches?* Pl. II. Fig. 3.

Then $(3{,}1416 \times 30 \times 60 =)$ 5654,88, the convex superficies.

EXAMPLE II.

What is the superficies of a cylinder whose circumference is 100 *inches, and the length* 14 *feet?*

Now 100 inches $= 8{,}\cancel{3}$ feet; and $8{,}\cancel{3} \times 14 = 116{,}\cancel{6}$ feet the curve superficies.

And $8{,}\cancel{3} \times 8{,}\cancel{3} \times 0{,}07958 \times 2 = 11{,}052\cancel{7}$, the area of both the ends.

Then, $116{,}\cancel{6} + 11{,}052\cancel{7} = 127{,}719\cancel{4}$, the whole superficies.

PRO-

PROPOSITION VI.

To find the convex superficies of a right cone; the $\left\{\begin{array}{l}\textit{diameter}\\ \textit{circumf.}\end{array}\right\}$ of whose base or end, and the length of the slant side being given.

RULE.

Multiply the $\left\{\begin{array}{l}\text{diam. by } 1,5708 \\ \text{circu. by } 0,5\end{array}\left(=\frac{p}{2}\right)\right\}$ and the product by the length of the side, gives the convex superficies.

EXAMPLE I.

What is the convex superficies of a right cone, the length of the slant side (AC) *being* 50; *and the diameter of the base* (AB) 20? Pl. II. Fig. 5.

Then ($1,5708 \times 20 \times 50 =$) 1570,8 is the convex surface.

EXAMPLE II.

What is the convex superficies of a right cone, whose circumference at the base is 24 *feet, and the length of the slant side* 32 *feet?*

Then ($24 \times 0,5 \times 32 =$) 384 is the convex superficies.

PROPOSITION VII.

In the frustum of a pyramid; whose ends are alike regular polygons, not exceeding 12 *sides; to find the solidity* (S) *thereof; the linear measures of* (AB, ab) *the two parallel ends, and their distance* (Cc) *being known.* Pl. II. Fig. 7.

RULE 1. Multiply one side of the greater end, by one side of the lesser end.

2. To the product add one third of the square of the difference of those sides.

3. Multiply the sum by the length.

4. Then this product multiplied by the polygon's factor (in tab. I. p. 144.) will give the solidity.

$$\text{Or } S = \overline{Ab \times ab \times \tfrac{1}{3}\,\overline{AB\ ab}^{2}} \times Cc \times N.$$

EXAMPLE I.

What is the solidity of the frustum of a square pyramid, one side of the greater end being 18 *inches; that of the lesser end,* 15 *inches, and the height* 60 *inches?*

Now

Now $18-15=3$ the difference of the fides.

And $\frac{3\times 3}{3}$ the third part of the fquare of that difference.

Then $(\overline{18\times 15+3}\times 60=)$ 16380 is the folidity required.

EXAMPLE II.

What is the folidity of the fruftum of a hexagonal pyramid; the fide of whofe greater end is 3 feet; that of the leffer end, 2 feet; and the length 12 feet?

Now $3-2=1$, and $\frac{1\times 1}{3}=0{,}3$ is $\frac{1}{3}$ of the fquare of the difference of the fides.

Then $\overline{3\times 2+0{,}3}\times 12\times 2{,}598076$ gives 197, 453776 for the folidity required.

If the ends of a pyramid's fruftum be of any other figure than that of a regular polygon; find the fide of a fquare, whofe area fhall be equal to the area of the greater end, and do the fame for the leffer end; then work as the foregoing rule directs.

PROPOSITION VIII.

To find the solidity (S) *of the frustum of a right cone, the length or height* (Cc), *and the* $\left\{\begin{array}{l}\textit{diameter (AB,ab)}\\ \textit{circumference.}\end{array}\right\}$ *at each end being given.*

Pl. II. Fig. 8.

RULE.

1. Multiply the $\left\{\begin{array}{l}\text{diam.}\\ \text{circu.}\end{array}\right\}$ at one end, by that at the other.

2. To the prod. add the squs. of those $\left\{\begin{array}{l}\text{diameters.}\\ \text{circumfes.}\end{array}\right.$

3. Multiply the sum by the given length.

4. The product multip. by the N°. $\left\{\begin{array}{l}0{,}2618\\ 0{,}026526\end{array}\right.$ will give the solidity.

$$\text{Or } S = \overline{AB \times ab + AB^2 + ab^2} \times Cc \times \frac{p}{12}.$$

EXAMPLE I.

What is the solidity of the frustum of a cone, the diameter of the greater end being 4 feet; that of the lesser end, 2 feet; and the altitude 9 feet?

Now $4 \times 2 = 8$, the product of the two diameters.
And $4 \times 4 = 16$: $2 \times 2 = 4$: $8 + 16 + 4 = 28$.
Then $28 \times 9 \times 0{,}2618 = 65{,}9736$ feet, the solidity.

EX-

EXAMPLE II.

What is the solidity of the frustum of a cone; the circumference of the greater end being 40; *that of the lesser end* 20; *and the length or height* 50?

Now $40 \times 20 + 40 \times 40 + 20 \times 20 = 2800$.

Then $(2800 \times 50 \times 0{,}026526 =)\ 3713{,}64$ is the solidity required.

Note, The Rule of Prop. VII. will find the solidity of a frustum of a cone, by using the diameters of the ends of the cone, instead of the sides of the pyramid; and changing the 4th article, for that in the Rule of Prop. VIII.

And the Rule of Prop. VIII. will serve for Prop. VII. by making a like change.

By these two last propositions, should all unequal squared or round timber be measured; for most trees, being bigger at the ground end than at the other, may be considered (when first cut down, and the branches lopt off) as the frustums of cones; and if the sides are cut rectangular, they then may be considered as the frustums of pyramids; and consequently, in either case, they should be measured according to the figure they represent, supposing them to be regular; but if the difference of the ends be small, there is no need of having recourse to any other directions than those given in the first proposition; and as all pyramids and cones, are considered as having their sides perfectly strait, most trees will differ from them, on account of the inequality of their sides or girths.

PRO-

PROPOSITION IX.

To find the convex surface of the frustum of a right cone, the length of the slant side, and the $\left\{\begin{array}{l}\textit{diameters}\\ \textit{circumf.}\end{array}\right\}$ *of the parallel ends being given.*

RULE.

Multiply the sum of the $\left\{\begin{array}{l}\text{diameters by } 1,5708\\ \text{circumfe. by } 0,5\end{array}\right\}$ the product multiplied by the length of the side will give the solidity.

EXAMPLE I.

What is the convex surface of the frustum of a right cone; the diameters of the ends being 8 *and* 4 *feet, and the length of the side* 20 *feet?*

Now $8+4=12$ the sum of the diameters.

And $12 \times 1,5708 \times 20 = 376,992$ the convex surface.

EXAMPLE II.

What is the convex surface of the frustum of a right cone; the circumference of the greater end being 30 *feet, that of the lesser end* 10 *feet; and the length of the slant side* 20 *feet?*

Then $(\overline{30+10} \times 0,5 \times 20 =)$ 400 feet is the convex surface required.

PRO-

PROPOSITION X.

To find the length or height, (CD) *of a pyramid or cone, the linear dimensions* (AB, ab, Cc) *of a frustum of that pyramid or cone being known.* Pl. II. Fig. 7. 8.

RULE.

Multiply the height of the frustum by the $\left\{\begin{matrix}\text{side}\\ \text{dia.}\end{matrix}\right\}$ of the greater end.

Divide the product by the diff. of the $\left\{\begin{matrix}\text{sides}\\ \text{dias.}\end{matrix}\right\}$ of the two ends; and the quotient will be the height of the $\left\{\begin{matrix}\text{pyramid.}\\ \text{cone.}\end{matrix}\right.$

Or $CD = \dfrac{Cc \times AB}{AB - ab}$.

EXAMPLE I.

In a frustum of a square pyramid; one side of the greater end is 5 feet; one side of the lesser end is 3 feet; and the distance of the ends is 8 feet: What is the height of that pyramid?

Now $5 - 3 = 2$ the difference of the sides.

Then $\left(\dfrac{5 \times 8}{2} = \right)$ 20 is the height of the pyramid.

EXAMPLE II.

There is a frustum of a cone; the diameter of the greater end is 83 inches; that of the lesser end is 54 inches; and the altitude is 12 feet; what is the altitude of the cone?

Now $83 - 54 = 29$ inches, which is 2,416 feet, the difference of the diameters; and 83 inches = 6,916 feet.

Then $\dfrac{6{,}916 \times 12}{2{,}416} = 34{,}3448$ is the height of the cone as required.

PRO-

PROPOSITION XI.

The dimensions of a pyramid or cone being given; to find what length from the vertex will answer to a given part of the solidity.

RULE.

Say as the solidity of the whole pyramid or cone, is to the cube of it's altitude; so is any given part of the solidity, to the cube of it's altitude, reckon'd from the vertex downwards; whose cube root is the length.

EXAMPLE.

There is a conical piece of timber, the diameter of whose base is 18 *inches, and the length* 12 *feet: What distance from the vertex must the saw be applied, to cut it into two pieces of equal solidity?*

Now 18 inches $=$ 1,5 feet.

And $1,5 \times 1,5 \times 12 \times 0,2618 = 7,0686$ feet the solidity; half thereof, is 3,5343.

And $12 \times 12 \times 12 = 1728$ the cube of the length.

Then 7,0686 : 1728 : : 3,5343 : 864, whose cube root is 9,5244; and so far from the vertex, must the saw be applied.

PRO-

PROPOSITION XII.

In a prismoid, having given its length or height, and also the length and breadth of each end; to find the solid content.

Note, A prismoid is a solid contain'd under six planes; whereof the parallel ends are unlike rectangles; and of the other four sides, each opposite two, are equal trapeziums.

RULE.

Multiply the length at the greater end, by the breadth at the lesser end; and the length at the lesser end, by the breadth at the greater end.

To half the sum of these two products, add the areas of the two ends.

The sum multiplied by a third of the height, gives the solidity.

Or, To the breadth of the greater end, add half the breadth at the lesser end, multiply the sum by the length at the greater end.

To the lesser breadth, add half the greater breadth, multiply the sum by the lesser length.

The sum of these two products, multiplied by a third of the height, gives the solidity.

EXAMPLE.

What is the solid content of a prismoid, whose greater end measures 12 *inches by* 8; *the lesser end*, 8 *inches by* 6; *and the length, or height*, 60 *inches?*

Now $12\times6=72$; $8\times8=64$; and $\frac{72+64}{2}=68$.

Then $\overline{68+12\times8+8\times6}\times20=4240$, the solidity.

Or, $\overline{8+\frac{6}{2}}\times12=132$; and $\overline{6+\frac{8}{2}}\times8=80$.

Then $\overline{132+80}\times20=4240$ is the solidity.

PRO-

PROPOSITION XIII.

To find the solidity of the Hoofs (aefb AE, FB AE*fb) of the frustum of a pyramid, the linear measures of the end, and the length being known.* Pl. II. Fig. 7.

If it is of any other form than a square pyramid, reduce it to such, by finding the side of a square of equal area; then,

RULE.

To the square of the side of one end, add one half the product of the sides of the two ends; this sum, multiply'd by one third of the height, gives the solidity.

Note, The solidity of the greater or lesser ungula will be obtain'd, according to the end of the frustum used.

Or greater hoof $= \overline{\overline{AB}^2 + \frac{1}{2}\overline{AB} \times ab} \times \frac{1}{3}Cc.$

lesser hoof $= \overline{\overline{ab}^2 + \frac{1}{2}\overline{AB} \times ab} \times \frac{1}{3}Cc.$

EXAMPLE.

There is a frustum of a square pyramid, one side of the greater end is 1,5 feet; the side of the lesser end is 1,25 feet, and the height is 5 feet; what is the solidity of the ungula?

Now

Now $1,5 \times 1,5 = 2,25$, the square of the greater side.

And $\frac{1,5 \times 1,25}{2} = 0,9375$.

Then $(\overline{2,25 + 0,9375} \times \frac{5}{3} =) 5,3129$ is the solidity of the greater hoof.

Again, $\overline{1,25 \times 1,25} = 1,5625$.

Then $(\overline{1,5625 + 0,9375} \times \frac{5}{3} =) 4,16$ is the solidity of the lesser ungula.

PROPOSITION XIV.

In a conical frustum, the diameters, and distance of the two parallel ends being given; to find the solidity of each hoof, when the frustum is cut by a diagonal plane. (Fig. 8.)

For the greater hoof (S).

RULE.

From the cube of the greater diameter, take the square root of the product of the cubes of the two diameters.

Divide the remainder by the difference of the two diameters.

Multiply the quotient by the height.

The product multiplied by 0,2618, will give the solidity.

$$\text{Or } S = \frac{\overline{AB}^3 - \sqrt{\overline{AB}^3 \times \overline{ab}^3}}{AB - ab} \times Cc \times \frac{p}{12}.$$

For

For the lesser hoof (s).

RULE.

From the square root of the product of the cubes of the two diameters, take the cube of the lesser diameter.

Divide the remainder by the difference of the two diameters.

Multiply the quotient by the height.

The product multiplied by 0,2618 will give the solidity.

$$\text{Or } S = \frac{\sqrt{\overline{AB}^3 \times \overline{ab}^3} - \overline{ab}^3}{AB - ab} \times Cc \times \frac{p}{12}.$$

EXAMPLE.

There is a conical frustum, the diameter of the greater end is 4 feet; that of the lesser 2 feet; and the height 9 feet; what is the solidity of each hoof?

Now $(4 \times 4 \times 4 =)$ 64 is the cube of the greater diameter.

And $(2 \times 2 \times 2 =)$ 8 is the cube of the lesser diameter.

Also $64 \times 8 = 512$; whose square root is 22,6272.

Then $\left(\frac{64 - 22{,}6272}{4 - 2} \times 9 \times 0{,}2618 = \right)$ 48,7413 is the solidity of the greater hoof.

And $\left(\frac{22{,}6272 - 8}{4 - 2} \times 9 \times 0{,}2618 = \right)$ 17,2323 is the solidity of the lesser hoof.

SEC-

SECTION IV.

Of a sphere and its parts.

PROPOSITION XV.

To find the superficies of a sphere or globe.

I. *The diameter being known.*

Rule. Multiply the square of the diameter by 3,1416; $(=p)$ and the product is the superficies.

Ex. *What is the superficies of a sphere, whose diameter is $1\frac{1}{3}$ feet?*

Then $1,3 \times 1,3 \times 3,1416 = 5,58506$ feet is the superficies.

II. *The circumference of a circle bisecting that sphere being known.*

Rule. Multiply the square of the circumference by 0,31832; $\left(=\frac{1}{p}\right)$ and the product is the superficies.

Ex.

Ex. *What is the superficies of a sphere, whose circumference is* 4,1888 *feet?*

Then $(4,1888 \times 4,1888 \times 0,31832 =)$ 5,5852 feet is the superficies.

III. *The diameter and circumference being known.*

RULE. Multiply the diameter by the circumference, and the product is the superficies.

Ex. *What is the superficies of a sphere, whose diameter is* $1\frac{1}{3}$ *feet, and the circumference is* 4,1888 *feet?*

Then $(1,\dot{3} \times 4,1888 =)$ 5,58506 is the superficies.

PROPOSITION XVI.

To find the solidity of a sphere.

I. *The diameter being known.*

RULE. Multiply the cube of the diameter by 0,5236 $\left(=\frac{p}{6}\right)$ and the product is the solidity.

Ex. *What is the solidity of a sphere, whose diameter is* $1\frac{1}{3}$ *feet?*

Then

Then ($1,3 \times 1,3 \times 1,3 \times 0,5236 =$) 1,2411 feet is the solidity.

II. *The circumference of a circle bisecting that sphere being known.*

Rule. Multiply the cube of the circumference by 0,016887; $\left(= \frac{1}{6pp}\right)$ and the product is the solidity.

Ex. *What is the solid content of a sphere, whose circumference is 4,1888?*

Then ($4,1888 \times 4,1888 \times 4,1888 \times 0,016887 =$) 1,2411 is the solid content.

If the superficies and diameter are known.

Rule. Multiply the superficies by one sixth of the diameter, and the product is the solidity.

Ex. *What is the solidity of a sphere whose diameter is* $1\frac{1}{3}$ *feet, and the superficies* 5,58506 *feet?*

Then $\left(5,58506 \times \frac{1,3}{6} =\right)$ 1,2411 feet is the solidity.

IV.

IV. *If the superficies and circumference are known.*

RULE. Multiply the superficies by the circumference, and the product multiply'd by 0,05305 $\left(=\frac{1}{6}p\right)$ gives the solidity.

EX. *What is the solidity of a sphere whose superficies is 5,58506, and whose circumference is 4,1888?*

Then ($5,58506 \times 4,1888 \times 0,05305 =$) 1,2411 is the solidity.

Note, The solidity of a sphere, is equal to two thirds of its circumscribing cylinder.

PROPOSITION XVII.

The superficies of a sphere being known.

I. *To find the diameter.*

RULE. Multiply the square root of the superficies by 0,56419 $\left(=\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter.

EX. *What is the diameter of that sphere, whose superficies 5,5852 feet?*

Now $\sqrt{5,5852} = 2,3633$.

Then ($2,3633 \times 0,56419 =$) 1,3 feet is the diameter required.

II.

II. *To find the circumference of a circle bisecting that sphere.*

RULE. Multiply the square root of the superficies by 1,77245, ($=\sqrt{p}$) and the product will be the circumference.

Ex. *If the superficies is 5,5852; what is the circumference?*

Then ($\sqrt{5,5852} \times 1,77245 =$) 4,1888 is the circumference.

III. *To find the solidity.*

RULE. Multiply the superficies, its square root, and 0,0940316 $\left(=\frac{1}{6}\sqrt{\frac{1}{p}}\right)$ continually, the product will give the solidity required.

Ex. *The superficies* = 5,5852; *required the solidity?*

Then ($5,5852 \times \sqrt{5,5852} \times 0,0940316 =$) 1,2411 is the solidity sought.

PROPOSITION XVIII.

The solidity of a sphere being known,

I. *To find the diameter.*

RULE. The cube root of the solidity, multiplied by 1,2407 $\left(=\sqrt[3]{\frac{6}{p}}\right)$ will give the diameter.

Ex. *Required the diameter of that sphere whose solidity is* 1,2411?

Now $\sqrt[3]{1,2411} = 1,074655$.
Then $(1,074655 \times 1,2407 =)$ 1,3 is the diameter.

II. *To find the circumference of a circle bisecting that sphere.*

RULE. Multiply the cube root of the solidity by 3,897777 $(= \sqrt[3]{6pp})$ the product will be the circumference.

Ex. *If the solidity is* 1,2411; *required the circumference?*

Then $(\sqrt[3]{1,2411} \times 3,897777 =)$ 4,1888 is the circumference sought.

III.

III. *To find the superficies.*

Rule. The cube root of the solidity multiplied by itself, and the product by 4,835976 ($=\sqrt[3]{36p}$) will give the superficies required.

Ex. *If the solidity is 1,2411; what is the superficies?*

Now $\sqrt[3]{1,2411} = 1,074655$.

Then ($1,074655 \times 1,074655 \times 4,835976 =$) 5,58498 is the superficies sought.

PROPOSITION XIX.

To find the convex superficies (s) *of a segment of a sphere cut off by a plane.*

I. *The diameter* (BA) *of the sphere, and the height* (DE) *of the segment* (FEG) *being known.* Pl. II. Fig. 10.

Rule. Multiply the diameter of the sphere by the height of the segment; then the product multiplied by 3,1416 will give the convex superficies.

Or $s = AB \times DE \times p$.

Ex. *What is the convex superficies of a segment whose height is* $4\frac{1}{2}$, *and cut from a sphere of* 21 *inches diameter?*

Then ($21 \times 4,5 \times 3,1416 =$) 296,8812, is the convex superficies.

II. *The diameter* (FG) *of the base of the segment, and its height* (DE) *being known.*

RULE. To the square of the diameter of the base, add the square of twice the height; the sum multiplied by 0,7854 will give the superficies required.

$$\text{Or } s = \overline{\overline{FG}^2 + \overline{2\,DE}^2} \times \frac{p}{4}.$$

Ex. *What is the convex superficies of that spherical segment, the diameter of whose base is* 17,23368, *and whose height is* 4,5?

Now 17,23368 × 17,23368 = 296,999, &c. or 297.

And $\overline{4,5 \times 2}|^2 = 81$.

Then $(\overline{297 + 81} \times 0,7854 =)$ 296,8812 is the superficies required.

PROPOSITION XX.

To find the solidity (S) *of a segment of a sphere.*

I. *The diameter* (FG) *of the base of the segment, and its height* (ED) *being known.* Pl. II. Fig. 10.

RULE. To thrice the square of (FD) half the diameter of the base, add the square of the height (DE,) multiply the sum by the height; then the product multiplied by 0,5236 will give the solidity.

$$\text{Or } S = \overline{3\,\overline{DF}^2 + \overline{DE}^2} \times DE \times \frac{p}{6}.$$

Ex.

Ex. *What is the solidity of a spherical segment, the diameter of the base being* 17,23368 *inches; and its height* 4,5?

Now $\frac{17,23368}{2} \times \frac{17,23368}{2} \times 3 = 222,75$.

And $4,5 \times 4,5 = 20,25$.

Then $\overline{222,75 + 20,25} \times 4,5 \times 0,5236 =$) 572,5566 is the solidity required.

II. *The diameter* (AB) *of the sphere, and the height* (DE) *of the segment being known.*

RULE. From thrice the diameter, subtract twice the height; multiply the remainder by the square of the height; the product multiplied by 0,5236 will give the solidity.

$$\text{Or } S = \overline{3\,AB - 2\,DE} \times \overline{DE}^2 \times \frac{p}{6}.$$

Ex. *In a sphere whose diameter is* 21; *what is the solidity of a segment thereof, whose height is* 4,5?

Now $\overline{21 \times 3} - \overline{4,5 \times 2} = 54$.

And $4,5 \times 4,5 = 20,25$.

Then ($54 \times 20,25 \times 0,5236 =$) 572,5566 is the solidity sought.

PROPOSITION XXI.

To find the convex superficies (s) *of a spherical zone or frustum.*

I. *The diameter* ($AB = d$) *of the sphere, and the breadth* ($MN = b$) *of the zone, or distance of the parallel ends being known.* Pl. II. Fig. 10.

Rule. Multiply the diameter of the sphere by the breadth of the zone; the product multiplied by 3,1416 ($= p$) will give the superficies required.

Or $s = dbp$.

Ex. *What is the convex surface of a spherical zone whose breadth is 4 inches; and cut from a sphere of 25 inches diameter?*

Then $25 \times 4 \times 3{,}1416 = 314{,}16$ the convex surface.

II. *Having the diameters* ($HI = c, KL = b$) *of the ends, and their distance* ($MN = b$) *given.*

Rule. Find the diameter of the sphere (by Prop. XXIII. Part I.) and then find the convex surface by the former rule.

Ex.

Ex. I. *In a ſpherical zone, the diſtance of whoſe parallel ends is 4 inches, the diameter of the greater end 24 inches, and that of the leſſer end, 20 inches: What is the convex ſurface, when the centre of the ſphere is without the zone?*

$$\text{Now } \frac{\frac{24}{2}\times\frac{24}{2}-\frac{20}{2}\times\frac{20}{2}+4\times4}{2\times4}=3{,}5 \text{ the diſ-}$$

tance of the greater end from the centre.

And $3{,}5\times2=7$.

Alſo $24\times24\times7+7=625$, whoſe ſquare root is 25.

Then $25\times4\times3{,}1416=314{,}16$ the convex ſurface.

Ex. II. *What is the convex ſurface of a ſpherical zone, the diſtance of whoſe parallel ends is 11 inches; the diameter of the greater end 24, and that of the leſſer end 20; the centre of the ſphere lying between the ends.*

$$\text{Now } \frac{\frac{20}{2}\times\frac{20}{2}+11\times11-\frac{24}{2}\times\frac{24}{3}}{11\times2}=3{,}5 \text{ the diſ-}$$

tance of the greater end from the center.

And $3{,}5\times2=7$.

Alſo $24\times24\times7+7=625$, whoſe ſquare root is 25, the diameter of the ſphere.

Then $25\times11\times3{,}1416=863{,}94$ ſquare inches, the convex ſurface.

III. *When the diameter* (HG=c) *and height* (DE=v) *of a segment cut from a hemisphere, is given; the convex surface* (s) *of the remainder* (HIBA) *may be thus found.* Pl. II. Fig. 10.

RULE.

1. Divide the fourth power of the diameter by thirty-two times the square of the height.

2. From the quotient, take half the square of the height.

3. The remainder multiplied by 3,1416 gives the convex surface.

$$\text{Or } s = \overline{\frac{c^4}{32\,vv} - \frac{vv}{2}} \times p.$$

Ex. *From a hemisphere, whose diameter is unknown, there is cut a segment, wherein the diameter of the base is 20 inches; and the height, or versed sine is 5 inches: Required the convex superficies of the remaining part?*

$$\text{Now } \frac{20\times 20\times 20\times 20}{32\times 5\times 5} = 200.$$

$$\text{And } 200 - \frac{5\times 5}{2} = 187{,}5.$$

Then $187{,}5 \times 3{,}1416 = 589{,}05$ the convex surface required.

PRO-

PROPOSITION XXII.

To find the solidity (S) *of a spherical zone, the radius* (MI$=c$,NL$=b$) *of each end, and their distance* (MN$=h$) *being known.*

RULE. To the square of the two radius's add one third of the square of the height: The sum multiplied by the height, and the product by 1,5708, gives the solidity.

$$\text{Or } S = \overline{cc + bb + \tfrac{1}{3}hh} \times h \times \frac{p}{2}.$$

Ex. *What is the solid content of a zone, whose greater diameter is* 24 *inches, that of the lesser* 20 *inches, and the height or distance of the ends is* 4 *inches?*

$$\text{Now } \overline{\frac{24}{2}}\Big|^2 + \overline{\frac{20}{2}}\Big|^2 + \frac{4 \times 4}{3} = 249{,}\dot{3}$$

Then $249{,}\dot{3} \times 4 \times 1{,}5708 = 1566{,}6112$ the solidity.

DEFINITION.

A *circular spindle*, is a solid described by the rotation of a circular segment about its chord: The circle of which the segment is a part, call, the *prime circle*; and the distance of its centre from that of the (middle of the chord, or) centre of the spindle, call the *central distance*.

PROPOSITION XXIII.

In the frustum (EFHG) *of a circular spindle* (AFBE); *wherein, the diameter* (FE $=2$D) *passing thro' the centre* (D), *a diameter* (HG $=2d$) *in another place, and the distance* (DI $=b$) *of those diameters are known; to find the central distance* (CD $=x$). Pl. II. Fig. 11.

RULE.

The difference of the squares of the half diameters, taken from the square of their distance, and the remainder divided by the difference of the diameters, gives the central distance.

$$\text{Or } x = \frac{hh - \overline{DD - dd}}{D - d}.$$

The central distance, added to half the greater diameter, gives the radius of the prime circle.

Ex. *In the frustum of a circular spindle, whose greatest diameter* EF $=36$, *lesser* GH $=16$; *and their distance* DI $=20$: *Required the central distance* CD, *and radius* CE *of the prime circle?*

Now $\left(\overline{20}\big|^2 - \overline{\overline{\tfrac{36}{2}}\big|^2 - \overline{\tfrac{16}{2}}\big|^2} \div \overline{36 - 16} = \right)$ 7 is the central distance.

And $\left(\frac{36}{2} + 7 = \right)$ 25 is the radius of the prime circle.

PRO-

PROPOSITION XXIV.

To find the solidity (S) *of a circular spindle; its length* (AB$=l$), *and greater diameter* (EF$=$D) *being known.*

RULE.

1st, Find the radius (CE$=r$) of the prime circle, and central distance (CD$=x$.) (Observing, that in this case, the lesser diameter$=$0.)

2d, Find the area (a) of a circular zone, whose breadth (AB) is equal to the length of the spindle; and the chord ($=$2CD) of each equal end, is equal to twice the central distance.

3d, Multiply the zone by the central distance, call the product A.

4th, To the square of the radius, add twice the square of the central distance, multiply the sum by two thirds of the length, call the product B.

5th, Then A taken from B, and the remainder multiplied by 3,1416, will give the solidity.

Or $\overline{\overline{rr + 2\,xx} \times \tfrac{2}{3}\,l. - ax} \times p = \mathrm{S}.$

EXAMPLE.

What is the solidity of a circular spindle, whose length is 48; and the greater diameter is 36?

Now $\left(\overline{\frac{48}{2}}\Big|^2 - \overline{\frac{36}{2}}\Big|^2 \div 36 =\right)$ 7 is the central distance.

And $\left(\frac{36}{2} + 7 =\right)$ 25 is the radius of the prime circle.

Also 636,611238 is the area of the generating circular segment.

Then $\left(\frac{\overline{7\times2}+\overline{7\times2}}{2} \times 48 + \overline{636,611238} \times 2 =\right)$ 1945,222476 is the area of the zone.

And $(1945,222476 \times 7 =)$ $13616,557332 = A$.

Also $(\overline{25}|^2 + 2 \times \overline{7}|^2 \times \frac{2}{3} \times 48 =) 23136 = B$.

Consequently $(B - A =) 9519,462668 \times 3,1416 =)$ 29906,2792 is the solidity required.

PRO-

PROPOSITION XXV.

To find the solidity, (S) of a frustum, of a circular spindle, wherein, one end (EF) passes thro' the centre (D): The diameters (EF,GH) of those ends, and their distance being known.

RULE.

Find the central distance ($CD = x$), and the radius ($CE = r$) of the prime circle,

Find the area (a) of a circular zone, wherein, one end, (or chord,) is equal to the frustum's lesser diameter added to twice the central distance (or $= 2\,CD + HG$); the other end, equal to the diameter of the prime circle; and the breadth, equal to the given distance (DI) of the diameters; multiply this zone by the central distance, call the product A. (Or put $A = xa$.)

To the square of the radius of the prime circle, add the square of the central distance; from the sum, take a third of the square of the length; multiply the remainder by the length; call the product B.

$$\text{Or } B = \overline{\overline{rr + xa} - \tfrac{1}{3}ll} \times l.$$

Then A taken from B, and the remainder multiplied by 3,1416 will give the solidity required.

EX-

EXAMPLE.

What is the solidity of a frustum of a circular spindle; the diameter of the end passing thro' the centre being 36; *that of the lesser end* 16; *and their distance* 20?

Now 7 is the central distance, and 25 is the radius.

And 879,5635 is the area of the zone.

Therefore $(879,5635 \times 7 =)$ 6156,9445 = A.

Also $(\overline{\overline{25}|^2 + \overline{7}|^2 - \frac{1}{3} \times \overline{20}|^2} \times 20 =)$ 10813,3 = B.

Therefore B—A = 4656,3888.

Then $(4656,3888 \times 3,1416 =)$ 14628,511 is the solidity sought.

This rule is of use in finding the content of such casks, as are in the form of circular spindles.

PROPOSITION XXVI.

To find the superficial content of a circular spindle, whose length and greatest diameter are known.

RULE.

Find the radius of the prime circle, and the central distance; and also, the length of the arc of the generating segment.

Multiply the radius of the prime circle by the length of the spindle; divide the product by the central distance; from the quotient subtract the length of the arc; multiply the remainder by twice the central distance; the product multiplied by 3,1416, will give the superficies required.

EXAMPLE.

What is the superficial content of a circular spindle whose length is 48; and its greatest diameter is 36?

Now 25 will be found for the radius; 7 for the central distance; and 64,34496 for the length of the arc.

Then $\left(\overline{\frac{25 \times 48}{7} - 64,34496} \times 7 \times 2 \times 3,1416 = \right)$ 4709,794 is the superficies required.

PRO-

PROPOSITION XXVII.

To find the convex superficies of the frustum of a circular spindle, wherein one end passes thro' the centre: The diameters of those ends, and their distance being known.

RULE.

Find the central diſtance, and the radius of the prime circle.

Seek the degrees (in tables) correſponding to a ſine found by dividing the given length of the fruſtum, by the radius of the prime circle; multiply theſe degrees by 0,0174534, and by the central diſtance; ſubtract the product from the length of the fruſtum; the remainder multiplied by the circumference of the prime circle, will give the convex ſurface required.

EXAMPLE.

What is the convex ſurface of the fruſtum of a circular ſpindle; the diameter of the end paſſing thro' the centre, being 36; *that of the leſſer end* 16; *and their diſtance* 20?

Now 7 is the central diſtance, and 25 is the radius of the prime circle.

And $\frac{20}{25} = 0{,}8$ is the ſine of 53,13 degrees.

Alſo $(25 \times 2 \times 3{,}1416 =)$ 157,08 is the circumfe.

Then $53{,}13 \times 0{,}0174534 \times 7 = 6{,}49103$.

Therefore $(20 - 6{,}49103 \times 157{,}08 =)$ 2121,989 is the ſuperficies ſought.

SEC-

SECTION V.

PRACTICAL QUESTIONS.

QUESTION I.

HOW many 3 inch cubes may be cut out of a 12 *inch cube?*

Now $\frac{12}{3} = 4$ the number of 3 inches in one side of a 12 inch cube.

Then $4 \times 4 \times 4 = 64$, the number of 3 inch cubes contain'd in a 12 inch cube.

QUESTION. II.

A farmer borrow'd of his neighbour a piece of a hayrick, which measured 6 feet every way; (that is, a cube whose side was 6 feet,) and the borrowing farmer, paid back two equal cubical pieces, each of whose sides were three feet; Query, whether the lending farmer was fully paid?

Now $6 \times 6 \times 6 = 126$, the solidity of the piece borrowed.

And $3 \times 3 \times 3 = 27$.

Therefore $27 \times 2 = 54$, the solidity of the pieces paid.

Then $\frac{216}{54} = 4$; therefore the lending farmer was paid but a fourth.

QUES-

QUESTION III.

One bespoke an iron roller for a garden, the outside diameter was to be 20 inches; length of the roller, 50 inches; and thickness of the metal, $1\frac{1}{2}$ inch; now supposing every cube inch weighs $4\frac{1}{4}$ ounces; what will the whole come to at $3\frac{1}{4}d.$ per lb?

Now $1\frac{1}{2} \times 2 = 3$, the double thickness; and $20 - 3 = 17$, the inner diameter.

Then $20 \times 20 \times 0{,}7854 = 314{,}16$; and $17 \times 17 \times 0{,}7854 = 226{,}9806$.

Therefore $314{,}16 - 226{,}9806 = 87{,}1794$.

And $87{,}1794 \times 50 = 4358{,}97$ the solidity of the roller.

But $4\frac{1}{4}$ *oz.* $= 0{,}265625$ lb.

Therefore 1 *inch.* : 0,265625 lb : : 4358, 97 *inch.* : 1157,8514, *&c.* the weight.

And $3\frac{1}{4}d. = 0{,}0135416$ £.

Therefore 1 lb : 0,0135416 £. : : 1157,8514 : 15,679 £. *&c.* = 15£. 13s. 7d. the whole value of the roller.

QUES-

QUESTION IV.

A Mason has set up two stone lamp-posts each consisting of a 16 *inch square pyramidal shaft of* 7 *feet high; and a parallelopiped pedestal of* 18 *inches square, and* 3 *feet high: What will they come to, at* 2*s*. 6*d*. *a foot?*

Now 18 inches $=$ 1,5 feet, one side of the base of the pedestal.

And 1,5 $\times$ 1,5 $\times$ 3 $=$ 6,75 feet, the solidity of one pedestal.

Also 16 inches $=$ 1,3 feet.

Now 1,3 $\times$ 1,3 $\times \frac{7}{3}$ $=$ 4,148, the solidity of one pyramid.

Then 6,75 $+$ 4,148 $=$ 10,89814 the solidity of one post.

And 10,89814 $\times$ 2 $=$ 21,7963 the solidity of both

Now 2*s*. 6*d*. $=$ 0,125£.

Then 1 f. : 0,125£. : : 21,7963 : 2,724£.

Or 2£. 14*s*. 6*d*. the whole expence.

QUESTION V.

A, has a cone of marble, the circumference of whose base is 37,6992 inches, and the height 24; which he would exchange with B, for a fine porphyry octagonal pyramid of the same height; and each side of the base is 5 inches: B insists to be paid 3 s. an inch for the difference of the solidities: What sum will he receive?

Now $37{,}6992 \times 37{,}6992 \times 0{,}026529 \times 24 =$ 904,7885 the solidity of the cone.

And $5 \times 5 \times 4{,}828427 \times \frac{24}{3} = 965{,}6854$ the solidity of the pyramid.

Therefore $965{,}6854 - 904{,}7885 = 60{,}8969$ the difference of the solidities.

Then if 1 inch : 0,15 £. : : 60,8969 : 9,1545 £.

Or 9 £. 2 s. $8\frac{1}{2}$ d. the sum *A* must give to boot.

QUESTION VI.

What will the painting a conical church spire come to at 8 d. per yard; supposing the circumference of the base 64 feet, and the altitude 118 feet?

Now 1 : 0,318309 : : 64 : 20,371776, the diameter of the base.

And $\frac{20{,}371776}{2} = 10{,}185888$, the radius.

Again, $118 \times 118 = 13924$; and $10{,}185888 \times 10{,}185888 = 103{,}7523$, &c.

Then $13924 + 103{,}7523 = 14027{,}7523$; whose square root is 118,43 feet, the slant side.

Therefore

Therefore 118,43 × $\frac{64}{2}$ = 3789,76 feet; and $\frac{3789,76}{9}$ = 421,084 yards.

Now 1 *yd.* : 0,03 *£.* : : 421,084 *yds.* : 14,03614 *£.* = 14 *£.* 0 *s.* 8¾ *d.* and so much will be the whole expence.

QUESTION VII.

One has in his garden, a regular octagonal pyramid, of 7 feet high, with a cubical dial fixed to its vertex; one side of the cube is equal to one side of the pyramid's base, which is 9 inches; what will the gilding this pyramid and cube come to, at 2 d. a square inch?

To solve this question.

1st, Find the length of a line drawn from the centre of the base, to the middle of one of its sides.

2dly, Find the length of a line drawn from the vertex of the pyramid, to the middle of one side in the base, this line will be the perpendicular height of one of the eight triangles. The rest is obvious.

Now 9 × 1,2071068 (tab. I. p. 144.) = 10,863969 is the radius of the inscrib'd circle; or equal to the distance of the centre of the base from the middle of one side.

And 7 feet = 84 inches.

Then $\sqrt{\overline{10,863965}|^2 + \overline{84}|^2}$ = 84,7 is the slant side of the pyramid, or height of one of the triangles composing the pyramid.

Then

Then $84,7 \times \frac{9}{2} \times 8 = 3049,2$ the pyramid's superficies.

Alſo $9 \times 9 \times 6 = 486$ the ſuperficies of the cube.

Then $3049,2 + 486 = 3535,2$ the ſuperficies of the pyramid and cube.

And 1 inch. : 0,0083£. (= 2 *d.*) : : 3535,2 : 29,46£.

Or 29£. 9*s.* 2½*d.* the whole expence.

QUESTION VIII.

What will a marble fruſtum of a cone come to, at 12s. a foot ſolid; the diameter of the greater end being 4 feet; that of the leſſer end, 1½ feet; and the length of the ſlant ſide 8 feet.

Now $\frac{4-1,5}{2} = 1,25$.

And $1,25 \times 1,25 = 1,5625$.

Alſo $8 \times 8 = 64$.

Then $64 - 1,5625 = 62,4375$, whoſe ſquare root is 7,9 feet, the altitude of the fruſtum.

Alſo $4 \times 4 = 16$; $1,5 \times 1,5 = 2,25$; $4 \times 1,5 = 6$.

And $16 + 2,25 + 6 = 24,25$.

Then $24,25 \times 7,9 \times 0,2618 = 50,154335$ feet, the ſolidity of the fruſtum.

There. 1*f.* : 0,6£. : : 50,154335f. : 30,092601£. = 30£. 1*s.* 10¼*d.* the expence of the whole.

QUES-

QUESTION IX.

Suppose a church-spire was to be built of an octagonal form; one side of the greater end to be 22 feet; one side of the lesser end, to be 12,5 feet; and the height 80 feet; but the inside of the spire is to be run up in a conical form; the diameter of the base is to be 56 feet; and the diameter at top to be 28 feet; What will be the expence, at 4s. 6d. per foot solid.

Now $12,5 \times 24 = 300$, the product of the sides of the ends.

And $24 - 12,5 = 11,5$ and $11,5 \times 11,5 = 132,25$.

Consequently $\frac{132,25}{3} = 44,083$ = to $\frac{1}{3}$ of the square of the difference of the two sides.

Then $\overline{300 + 44,083} \times 80 \times 4,828427 = 132910,5$ the solidity of the frustum of the pyramid.

Again $56 \times 28 = 1568$; $56 - 28 = 28$; $28 \times 28 = 784$; $\frac{784}{3} = 261,3$; and $1568 + 261,3 = 1829,3$.

Then $1829,3 \times 80 \times 0,7854 = 114940,672$, the solidity of the contain'd cone.

Therefore $132910,5 - 114940,672 = 17969,828$ feet, the solidity of the stone work.

Then 1 f. : 0,225£. :: 17969,828f. : 4043,2113£. = 4043£. 4s. 2¾d. the cost.

QUESTION X.

A mason is employ'd to complete a decayed Portland stone cone; he having made the upper part level, the measures of the frustum are as follows: length of the slant side 12 feet; diameter of the upper end 6 feet; and the circumference of the base 38 feet; What will it cost at 3s. per foot, to put a piece on, equal to that taken of?

Now 1 inch : 0,318309 : : 38 : 12,095742, or 12,1, the diameter of the base.

And $12,1 - 6 = 6,1$.

Therefore $\frac{6,1}{2} = 3,05$, the half difference of the diameters.

Then $12 \times 12 = 144$; $3,05 \times 3,05 = 9,3025$; and $144 - 9,3025 = 134,6975$, whose square root is 11,6, &c. the frustum's height.

Then 6,1 : 11,6 : : 6 : 11,4, &c. the height of the piece wanting.

And $6 \times 6 \times 0,7854 \times \frac{11,4}{3} = 107,44272$, &c. feet, the solidity of the piece wanting.

Then 1 f. : 0,15£. : : 107,44272f. : 16,1164, = 16£. 2s. 4d. the whole expence to compleat the cone.

QUES-

QUESTION XI.

Three men bought a tapering piece of timber, which was the frustum of a square pyramid; one side of the greater end was 3 feet; one side of the lesser end, 1 foot; and the length, 18 feet; and they paying equally, are to have equal shares; what is the length of each man's piece?

Now $\left(\overline{3\times 1+\frac{3-1\times 3-1}{3}}\times 18=\right)$ 78 the solidity of the frustum, (by Prop. VII.)

And $\left(\frac{78}{3}=\right)$ 26, the solidity of each man's share.

Also $\left(\frac{18\times 3}{3-1}=\right)$ 27 is the length of the pyramid. (by Prop. X.)

And $27-18=9$ is the length of the piece wanting.

Whose solidity is $\left(1\times 1\times \frac{9}{3}=\right)$ 3.

Therefore $78+3=81$ is the solidity of the pyramid.

Now $81:\overline{27}|^3::26+3:7047$; whose cube root is 19,172.

Also $81:\overline{27}|^3::3+26\times 2:13365$; whose cube root is 23,731.

Then $27-23{,}731=3{,}269$ for the length of the 1st man's share.

And $23{,}731-19{,}172=4{,}559$ the length of the 2d man's share.

And $19{,}172-9=10{,}172$ the length of the 3d man's share, reckoning the shares from the greater end towards the lesser.

QUESTION XII.

Two men purchase a piece of Egyptian granite, which is a conical frustum, with parallel ends; whose greater diameter is 50 inches; the lesser 20; and the length of the slant-side, 40 inches: And they agree to cut it, parallel to the ends, into two pieces of equal solidity; and he who takes the piece next to the greater end, shall pay to the other, 2 d. for every square inch difference between the whole superficies of the two pieces: How much will he receive, who takes the lesser end?

Now $\sqrt{\overline{40}\big|^2 - \overline{\frac{50-20}{2}}\Big|^2} = 37$ inches, is the height of the frustum.

And $30 : 37 :: 20 : 24,6$, the height of the piece wanting.

Th. $37 + 24,6 = 61,6$ the cone's height.

Also $\overline{50}\big|^2 \times \frac{61,6}{3} \times 0,7854 = 40360,83$ the cone's solidity.

And $\overline{20}\big|^2 \times \frac{24,6}{3} \times 0,7854 = 2583,093$, the solidity of the piece wanting.

Therefore $40360,83 - 2583,093 = 37777,74$ the solidity of the given frustum.

And $\frac{37777,74}{2} = 18888,87$ is the solidity of each man's share.

Also $40360,83 : \overline{61,6}\big|^3 :: 2583,093 + 18888,87 : 124756,454$ whose cube root is 49,9, &c. or 50 inches.

Then

Then ($50 - 24,6 =$) $25,3$ inches meaſured from the leſſer end, and parallel to the axis, will give the place in the fruſtum where the ſection is to be made.

And $61,6 : 50 :: 50 : 40,54$, the diameter at the ſection.

Now $37 : 40 :: 25,3 : 27,387$ the ſlant ſide of the upper fruſtum.

And $40 - 27,387 = 12,613$ the ſlant ſide of the lower fruſtum.

Alſo $\overline{50 + 40,54} \times 1,5708 \times 12,613 = 1793,8236$ the convex ſurface of the lower fruſtum.

And $\overline{20 + 40,54} \times 1,5708 \times 27,387 = 2604,4103$ the convex ſurface of the upper fruſtum.

Likewiſe $\overline{50}|^2 \times 0,7854 = 1963,5 =$ area at the greater end.

And $\overline{20}|^2 \times 0,7854 = 314,16 =$ area at the leſſer end.

Alſo $\overline{40,54}|^2 \times 0,7854 = 1290,7981 =$ area at the ſection.

Then $1793,8236 + 1963,5 + 1290,7981 = 5048,1217$ the ſuperficies of the lower fruſtum.

And $2604,4103 + 1290,7981 + 314,16 = 4209,3688$ the ſuperficies of the upper fruſtum.

Therefore $5048,1217 - 4209,3688 = 838,7529$ inches the difference of the ſuperficies. Which, at 2 *d.* an inch, amounts to 6,989 £. ; or 6 £. 19 *s.* 9½ *d.* the money he is to receive, who takes the fruſtum next the leſſer end.

QUESTION XIII.

Suppose the globe, or ball, on the top of St. Paul's church to be 6 f. in diameter; what did the gilding thereof come to at $3\frac{1}{2}$ d. per inch square?

Now 6 feet equal 7 inches.

And $72 \times 72 \times 3,1416 = 16286,0544$ the superficies.

Then 1 inch : ($3\frac{1}{2}$ *d.* =) 0,014583 £. : : 16286,0544 : 237,9935 £. ; or 237 £. 19 *s.* $10\frac{1}{2}$ *d.* the expence.

QUESTION XIV.

What is the weight of a bomb-shell, whose outside diameter is 16 inches; and the thickness of the metal, 3 inches; supposing a cubic inch weighs $4\frac{1}{2}$ ounces?

Now $16 - \overline{3 \times 2} = 10$ inches, the inside diameter.

And $16 \times 16 \times 16 \times 0,5236 = 2144,6656$.

Also $10 \times 10 \times 10 \times 0,5236 = 523,6$ the cubic inches in the concavity.

Therefore $2144,6656 - 523,6 = 1621,0656$ the solidity of the shell.

But $4\frac{1}{2}$ oz. = 0,28125 lb.

Then 1 : 0,28125 : : 1621,0656 : 455,9247 lb weight.

QUESTION XV.

A person wants a cylindric vessel of 3 feet deep, that shall hold twice as much as a vessel of 28 inches deep, and 46 inches in diameter throughout. What must be the diameter of the required vessel?

Now $\overline{46}|^2 \times 0,7854 \times 28 = 46533,3792$, the contents of the given vessel.

And $46533,3792 \times 2 = 93066,7584$, the contents of the required vessel.

Also $\frac{93066,7584}{36 (= 3f.)} = 2585,1877$, the area of the required vessel's base.

Then $1 : 1,2732 :: 2585,1877 : 3291,5$, whose square root is $57,37$ the diameter of the required vessel's base.

QUESTION XVI.

One has a grainery 47 feet 8 inches long, 18 feet 5 inches broad, and 9 feet 7 inches high, but wants another that will hold four times as much, and have the dimensions in the same proportion to each other as the old one has; What will be the length, breadth and depth of the new one?

Now 47 feet 8 inches $= 47,\dot{6}$; 18 feet 5 inches $= 18,41\dot{6}$; 9 feet 7 inches $= 9,58\dot{3}$.

Then $18,41\dot{6} \times 9,58\dot{3} \times 47,\dot{6} = 8412,836$, the solid content of the old Grainery.

And $8412,836 \times 4 = 33651,344$, the solid content of the new one.

Also $\overline{47,\dot{6}}^3 = 108303,9\dot{6}$, the cube of the length of the old one.

Then $8412,836 : 108303,9\dot{6} :: 33651,344 : 433215,866$, &c. whose cube root is 75,666, &c. the length of the new one.

And $47,\dot{6} : 75,\dot{6} :: 18,41\dot{6} : 29,23$, &c. the breadth of the new one.

Therefore $47,\dot{6} : 75,6 :: 9,58\dot{3} : 15,21$ nearly, the depth or height of the new one.

QUESTION XVII.

A gentleman is desirous of having in his park, a rectangular canal, of a quarter of a mile long, that shall contain 4 acres on the surface; be 7 feet deep; and the sides and ends to slope in the diagonal of a square whose side shall be the depth of the canal: What will the digging come to at 4 s. 6 d. a floor (of 18 feet square, and 1 foot deep;) for work and carriage?

An

An acre $= 4840$ square yards.

And 4 acres $= 19360$ square yards.

A quarter of a mile, is 440 yards.

Or 1320 feet, the length of the canal at the upper surface.

Then $\frac{19360}{440} = 44$ yards.

Or 132 feet for the breadth of the canal at the upper surface.

And $1320 - \overline{7 \times 2} = 1306$, the length at bottom.

And $132 - \overline{7 \times 2} = 118$, the breadth at bottom.

Then by Prop. 12.

$$\overline{132 + \frac{118}{2}} \times 1320 = 252120.$$

$$\text{And } \overline{118 + \frac{132}{2}} \times 1306 = 240304.$$

Then $\overline{252120 + 240304} \times \frac{7}{3} = 1148989,3$ feet the content of the canal.

And $\frac{1148989,3}{324 (= 18 \times 18)} = 3546,26$ floors.

Then 1 floor : 0,225 £. (= 4 *s.* 6 *d.*) : : 35462,62 : 797,0985.

Or 797 £. 18 *s.* 2 *d.* the expence.

QUESTION XVIII.

One who had been to gauge a vessel, which was the frustum of a cone, with parallel ends, and 9 feet long; found it held 404,2638 gallons of beer measure: And being asked for the diameters, said, he had forgot them, but that one was twice as much as the other: What were those diameters?

Note, A gallon, beer measure, is equal to 282 cubic inches.

Then $\frac{404{,}2638 \times 282}{1728} = 65{,}9736$ the solidity of the vessel in feet.

Now $\frac{65{,}9736}{9} = 7{,}3304$, the mean area, between the areas of the ends of the vessel.

And (as the area of a circle) 1 : (to the square of the diameter) 1,2732 £. : : (so is any other area =) 7,3304 : (to the square of the diameter) 9,3.

Between the squares of the terms 2 and 1, of the given ratio of the diameters, find a mean square.

Thus $\overline{2\times 1}+\frac{\overline{2-1}\times\overline{2-1}}{3} = 2{,}3$ the mean square.

Then (as any mean area =) 2,3 : (to the square of the greater extream =) 4 : : (so is any other mean area =) 9,3 : (to the square of its greater extream =) 16; whose square root 4, is the greater diameter.

And : 2 1 : : 4 : 2 feet the lesser diameter.

QUES-

QUESTION XIX.

Suppose two porters having a quart of strong beer between them, agree to drink it off at two pulls, that is, a draught to each; now the first having given it the black eye, as they call it, that is, drank till the surface of the liquor touch'd the opposite edge of the bottom, he gave the remaining part of it to the other; what was the difference of their shares? Supposing the quart pot was the frustum of a cone; the depth being 5,7 inches, the diameter at top 3,7 inches, and the solidity 70,5 solid inches?

First, The diameter of the bottom must be found, and then the difference of the solidities of the two hoofs.

Having given the solidity, length, and one of the diameters of a cone's frustum, the other diameter may be found by the following

RULE.

Multiply the length by 0,2618, and divide the given solidity by this product: From the quotient subtract $\frac{3}{4}$ of the square of the given diameter; from the square root of the remainder, take half the given diameter, and it will leave the diameter sought.

Or putting S = solidity; L = length; D = diam. sought; D = diam. given.

And $S : l \times \frac{p}{12} = A$.

Then $d \div \sqrt{A - \frac{3}{4}d.^2} - \frac{1}{2}\, d$.

 Now

Now $0,2618 \times 5,7 = 1,49226$.

And $\frac{70,5}{1,49226} = 47,243$.

Also $3,7 \times 3,7 \times \frac{3}{4} = 10,2675$.

And $47,243 - 10,2674 = 36,9755$ whose square root is, $6,08$, &c.

Then $6,08 - \frac{3,7}{2} = 4,23$ the diameter of the bottom of the pot.

Now $4,23 \times 4,23 \times 4,23 = 75,686967$.

And $3,7 \times 3,7 \times 3,7 = 50,653$.

Also $75,686967 \times 50,653 = 3833,771999$, whose square root is $61,917$.

Then $61,917 - 50,653 = 11,264$.

And $4,23 - 3,7 = 0,53$.

Also $\frac{11,264}{0,53} = 21,253$.

Then $21,253 \times 5,7 \times 0,2618 = 31,715$ cubic inches, what the first man drank.

And $70,5 - 31,715 = 38,785$ cubic inches, what the last man drank.

Then $38,785 - 31,715 = 7,07$ cubic inches, the difference of their shares.

PART

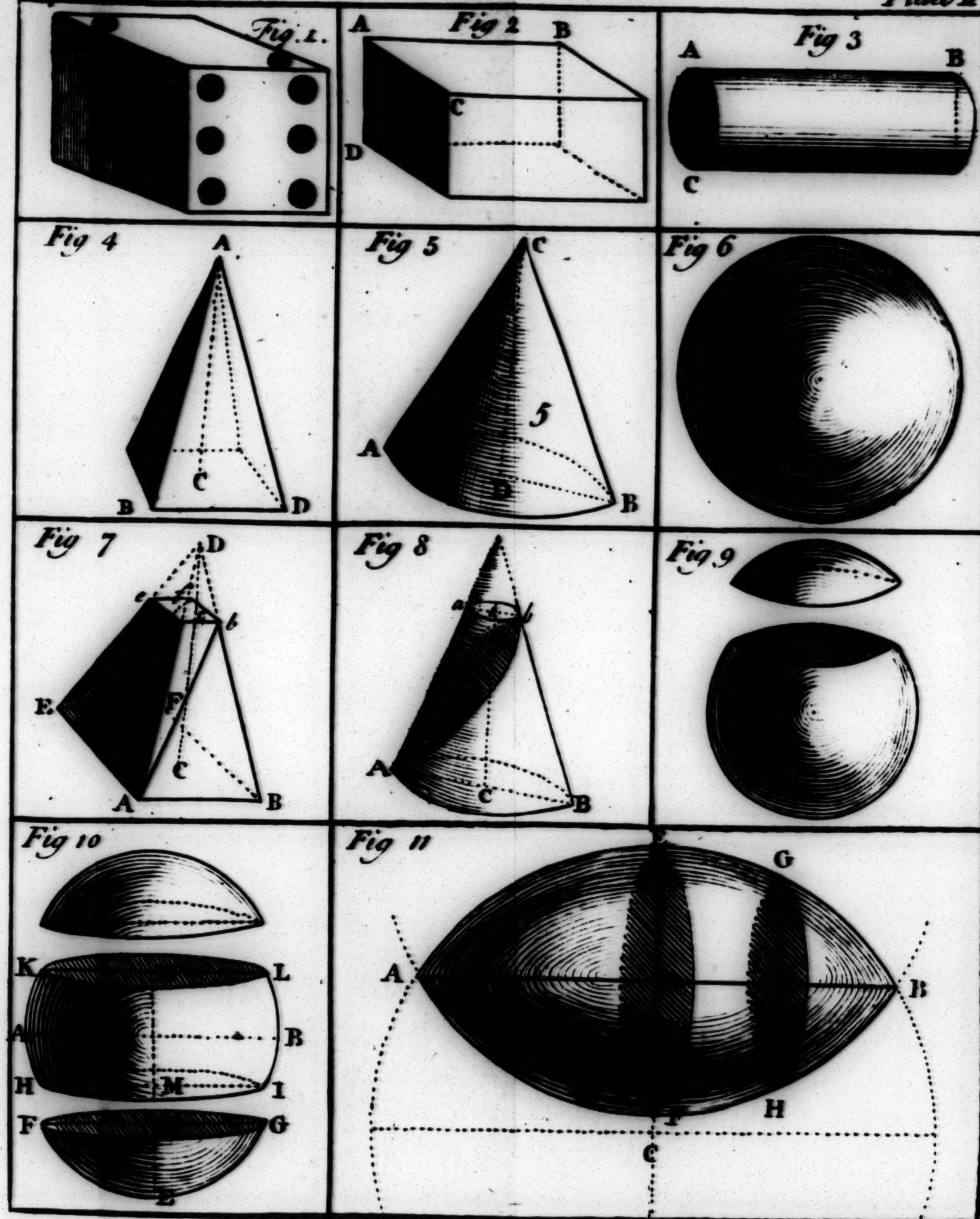
Fig. 1.
Fig 2
A
B
C
D
Fig 3
A
B
C
Fig 4
A
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Fig 5
C
5
A
D
B
Fig 6
Fig 7
D
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Fig 8
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Fig 9
Fig 10
K
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Fig 11
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C

PART III.

SECTION I.

I. IF a right cone (ABC) be cut by a plane (DE) parallel to its base, (BC), the section will be a *circle*. (Plate III. Fig. 1.)

II. If the plane (DF) pass thro' the opposite sides (AC, AB) of the cone, the section will be an *ellipsis*. (Fig. 1, 2.)

III. If the plane (DG) pass parallel to one side (AB) of the cone, thro' the opposite side (AC) and base (BC), the section will be a *parabola*. (Fig. 1, 3.)

IV. If the plane (DH) pass thro' one side (AC) of the cone, and the base (BC), in such a manner, as being continued upwards (in DI), would meet the opposite side (BA), of the cone continued upwards, (in AI), the section will be a *hyperbola*. (Fig. 1, 4.)

V. The point (D) of the curve, nearest the vertex (A) of the cone (ABC), and where the curve is most acute, is called *the principal vertex*. (Fig. 1.)

VI. A right line (AB) drawn thro' the principal vertex (A), dividing the area of the section into two equal parts, is called the *axis*. (Fig. 2, 3, 4.)

VII. In the ellipsis, this line (AB) is called the *transverse diameter*, or *transverse axe*. (Fig. 2.)

VIII. In the hyperbola, the continuation (DI), of the axe, till it meets the other side (BA), of the cone, continued, is called the *transverse diameter*. (Fig. 1.)

IX. A right line (PH=2PI), drawn at right angles to the axe (AB), and terminated at each end by the curve, is called a *double ordinate*. (Fig. 2, 3, 4.)

X. The double ordinate (DK) passing thro' the middle (C) of the transverse diameter (AB) in the ellipsis, is called the *conjugate diameter*, or *conjugate axe*. (Fig. 2.)

XI. That part (AI, or GD), of the axe (AB, or DK) intercepted between the curve and the ordinate (PI or GP), is called an *abscissa*. (Fig. 2, 3, 4.)

XII. A curved prismatic figure, with strait sides, and parallel equal elliptic ends, is called a *cylindroid*.

XIII. A figure, formed by the rotation of an ellipsis, round either of its axes, is called a *spheroid*: The fixed axis is called the *axe of rotation*; and the other axis is called the *revolving axe*.

XIV. If the transverse be the axe of rotation, the figure generated is called a *prolate spheroid*.

XV. If the conjugate be the axe of rotation, the figure generated is called an *oblate spheroid*.

XVI.

XVI. A figure generated by the rotation of a parabola, or a hyperbola, about its axe, is called a *conoid*.

XVII. If by a parabola, 'tis called a *parabolic conoid*, or *paraboloid*.

XVIII. If by a hyperbola, 'tis called a *hyperbolic conoid*, or *hyperboloid*.

Note, If a plane cut a spheroid, or conoid, oblique to the axe, the section will be elliptical.

XIX. A figure supposed to be generated by the rotation of a segment of an ellipsis, or of a parabola, about its ordinate, is called a *spindle*, and is denominated, either as *elliptic*, *parabolic*, or *hyperbolic*.

XX. A part of either of these solids, contained between two parallel sections, or plane ends, is called a *frustum*: And a part with only one plane end, is called a *segment*. (Fig. 7, 8, 12.)

SEC-

SECTION II.

Of elliptical lines, superficies, and solids.

PROPOSITION I.

In an ellipsis, any three of these four terms being given; viz. *half transverse,* (CB $=t$) *half conjugate,* (CD $=c$) *ordinate,* (FE $=y$) *abscissa,* (CE $=x$) *the other is easily found.* (Plate III. Fig. 2.)

Case I. *When the half transverse, half conjugate, and abscis are known, to find the ordinate.*

RULE.

From the square of the half transverse, take the square of the abscis; multiply the square root of the remainder by the quotient of the half conjugate, divided by the half transverse, and the product is the ordinate.

Or $y = \frac{c}{t}\sqrt{tt - xx}$.

EXAMPLE.

Suppose the half transverse (CB) *is* 60; *the half conjugate* (CD) *is* 20; *and the abscis* (CE) *is* 36: *Required the ordinate* EF?

Then $\left(\frac{20}{60} \times \sqrt{\overline{60}^2 - \overline{36}^2} = \right)$ 16 is the ordinate.

II.

II. *When the half transverse, half conjugate, and ordinate are known; to find the abscis.*

RULE.

From the square of the half conjugate, take the square of the ordinate; multiply the square root of the remainder, by the quotient of the half transverse, divided by the half conjugate, and the product is the abscis.

$$\text{Or } x = \frac{t}{c}\sqrt{cc - yy}.$$

EXAMPLE.

Suppose the half transverse (CB) *is* 60; *the half conjugate* (CD) *is* 20; *and the ordinate* (FE) *is* 16; *Required the abscis* (CE)?

Then $\left(\frac{60}{20} \times \sqrt{20 \times 20 - 16 \times 16} = \right)$ 36 is the abscis.

III. *The*

III. *The half conjugate, the ordinate, and absciss being known; to find the half transverse.*

RULE.

From the square of the half conjugate, take the square of the ordinate; make the square root of the remainder, a divisor to the product of the half conjugate by the absciss; and the quotient will be the half transverse.

$$\text{Or } t = \frac{cx}{\sqrt{cc-yy}}.$$

EXAMPLE.

Suppose the half conjugate (CD) *is* 20; *the ordinate* (EF) *is* 16; *and the absciss* (CE) *is* 36: *Required the half transverse* (CB)?

Then $\left(\frac{20\times 36}{\sqrt{20\times 20-16\times 16}}=\right)$ 60 is the half transverse.

IV. The

IV. *The half transverse, the ordinate, and the abscis being known; to find the half conjugate.*

RULE.

From the square of the half transverse, take the square of the abscis; let the square root of the remainder, be a divisor to the product of the half transverse by the ordinate; and the quotient will be the half conjugate.

Or $c = \frac{ty}{\sqrt{tt - xx}}$.

EXAMPLE.

Suppose the half transverse (CB) *is* 60; *the ordinate* (EF) *is* 16; *and the abscis* (CE) *is* 36: *Required the half conjugate?*

Then $\left(\frac{60 \times 16}{\sqrt{60 \times 60 - 36 \times 36}} =\right)$ 20 is the half conjugate.

PRO-

PROPOSITION II.

To find the periphery of an ellipsis, the transverse and conjugate diameters, or axes, being known.

RULE.

Multiply half the sum of the two diameters, by 3,1416; and the product will be the periphery, exact enough for most practical purposes.

EXAMPLE.

What is the periphery of an ellipsis, whose transverse axe is 24, *and the conjugate* 18?

Then $\left(\frac{24+18}{2} \times 3,1416 =\right)$ 65,9736 is the periphery sought.

The periphery of an ellipse may be found nearly accurate to five places, by the following table, constructed by Sir *Jonas Moore*; who says of it, "I have "made above 45000 arithmetical operations for "this table, and am now well pleased it is finished. "Some perhaps may find shorter ways, as I believed "I had myself, till advised otherwise by the truly "honourable the lord *Bruncker*. I therefore pursued the rules given by me, in contemplation "of the ellipsis printed in my arithmetic, taking "100 ellipses betwixt that which falls upon the "diameter (equal in this case to 2,0000, the first "in the table,) and the greatest which is the "circle, (the last.")

A TABLE

A Table for finding the Periphery of an Ellipsis.

A	Periph.	D.	A.	Periph.	D.	A.	Periph.	D.
1	2,0012	12	34	2,2368	104	67	2,6465	139
2	2,0028	16	35	2,2474	106	68	2,6604	140
3	2,0048	20	36	2,2581	107	69	2,6744	140
4	2,0072	26	37	2,2692	111	70	2,6884	141
5	2,0100	28	38	2,2803	111	71	2,7025	141
6	2,0133	33	39	2,2915	112	72	2,7166	143
7	2,0170	37	40	2,3028	113	73	2,7309	144
8	2,0213	43	41	2,3142	114	74	2,7453	146
9	2,0261	48	42	2,3256	114	75	2,7599	146
10	2,0314	53	43	2,3371	115	76	2,7745	146
11	2,0370	56	44	2,3488	117	77	2,7891	147
12	2,0432	62	45	2,3607	119	78	2,8038	148
13	2,0496	64	46	2,3726	119	79	2,8186	148
14	2,0564	68	47	2,3848	122	80	2,8334	148
15	2,0634	70	48	2,3970	122	81	2,8482	148
16	2,0708	74	49	2,4094	124	82	2,8630	149
17	2,0784	76	50	2,4218	124	83	2,8779	150
18	2,0862	78	51	2,4342	124	84	2,8929	150
19	2,0942	80	52	2,4467	125	85	2,9080	151
20	2,1024	82	53	2,4594	127	86	2,9231	151
21	2,1106	82	54	2,4723	129	87	2,9382	152
22	2,1192	86	55	2,4852	129	88	2,9534	152
23	2,1281	89	56	2,4983	131	89	2,9686	153
24	2,1373	92	57	2,5114	131	90	2,9839	154
25	2,1467	94	58	2,5245	132	91	2,9993	154
26	2,1561	94	59	2,5377	132	92	3,0147	155
27	2,1658	97	60	2,5510	133	93	3,0302	156
28	2,1756	98	61	2,5644	134	94	3,0458	156
29	2,1856	100	62	2,5779	135	95	3,0614	157
30	2,1956	100	63	2,5915	136	96	3,0771	157
31	2,2057	101	64	2,6052	137	97	3,0928	158
32	2,2160	103	65	2,6189	137	98	3,1086	158
33	2,2264	104	66	2,6327	138	99	3,1244	158

The

The use of the table, for finding the periphery of an ellipse, whose transverse and conjugate axes are known.

RULE.

Divide the conjugate by the transverse; if the quotient gives no more than two places, seek them in the columns signed *axe*; the number right against it, in the column signed *periphery*, multiplied by the given transverse, will give the periphery required.

EXAMPLE I.

Let the transverse be 1; and the conjugate 0,5: Required the periphery?

Now $\frac{0{,}5}{1} = 0{,}50$.

And against 50 (found in the column signed *axe*) is 2,4218; (found in the column signed *periphery*.) Then (2,4218 × 1 =) 2,4218 is the periphery sought.

EXAMPLE II.

Let the transverse be 24, and the conjugate 18: Required the periphery?

Now $\frac{18}{24} = 0{,}75$: Against it is 2,7599.

Then (2,7599 × 24 =) 66,2376 is the periphery required.

If

If the quotient of the conjugate by the transverse exceeds two places.

RULE.

1st. Seek the two first places of the quotient in the column signed *axe*, and take out the periphery against it; and also the number next below it, in the column signed *diff.*

2d. Multiply this diff. by the remaining part of the quotient; write the two left hand places under the two right hand places of the said periphery; their sum multiplied by the given transverse, will give the periphery required.

EXAMPLE III.

The longer axe 10000, *and the shorter axe* 4382: *Required the periphery?*

Now $\frac{4382}{10000} = 0,4382$.

Against 43, and under periph. is 2,3371.

Between 43 and 44, and under diff. is 117.

And $\underline{117 \times 82} = \underline{9504}$ or 96, or 0,0096.

Then $\overline{2,3371 + 0,0096} \times 10000 = 23467$ for the periphery required.

EX-

EXAMPLE IV.

Suppose the transverse diameter is 32,54, and the conjugate 18,64: Required the periphery?

Now $\frac{18,64}{32,54} = 0,57283$; against 57, and under periphery is 2,5114: and under diff. is 131;
And $131 \times 283 = 37073$, or 0,00037.
Then $(2,5114 + 0,0037 \times 32,54 =)$ 81,841 is the periphery.

PROPOSITION III.

To find the area of an ellipsis; the transverse and conjugate diameters, or axes, being known.

RULE.

Multiply the transverse by the conjugate diameter; the product multiplied by 0,7854 will give the area required.

EXAMPLE.

What is the area of an ellipsis whose transverse axe is 24; and the conjugate 18?

Then $(24 \times 18 \times 0,7854 =)$ 339,2928 is the area.

Note, The area of an ellipsis is a mean proportional between the area of a circle on its transverse axe, and the area of a circle on its conjugate axe.

PRO-

PROPOSITION IV.

In an elliptical segment, (PDF,PAH) *the base,* (PF,PH) *or double ordinate; the height,* (GD,IA$=$X) *or abscis; and, an abscis* (Dg,A$e=x$) *corresponding to an ordinate* (pg,pe) *equal to a fourth of the base,* (PF,PH) *being known; to find the length of* (DK,AB) *that axe of the ellipse to which the base is perpendicular.* Pl. III. Fig. 2.

RULE. From the square of the greater abscis, subtract four times the square of the lesser abscis; divide the remainder by the greater abscis lessened by four times the lesser abscis; and the quotient will shew the axe required.

Or, the axe $= \dfrac{XX - 4xx}{X - 4x}$

If the difference between the axe and abscis, multiplied by the abscis, be greater than the square of its corresponding ordinate; the axe found is the transverse; if less, the conjugate.

Ex. I. *In a segment of an ellipsis, the half base, or ordinate is* 40; *the height, or abscis, is* 12; *and the abscis to the ordinate* 20, *is* 2,50455: *Required the axe to which the ordinates are perpendicular?*

Then $\left(\dfrac{\overline{12}^2 - \overline{2,50455}^2 \times 4}{12 - 2,50455 \times 4} = 59{,}99, \&c. \text{ or}\right)$
60 the axe. (art. 13.)
Now $\overline{60-12} \times 12 = 576$; and $40 \times 40 = 1600$.
Therefore 60 is the conjugate axe.

Ex. II. *In an elliptic segment, whose abscis is* 10, *and ordinate* 18; *and the abscis to the ordinate* 9, *is* 2,30304: *Required the axe to which these ordinates are perpendicular?*

Then $\left(\frac{\overline{10}^2 - \overline{2,30304}^2 \times 4}{10 - 2,30304 \times 4} = \right)$ 100 is the axe.

Now $\overline{100-10} \times 10 = 900$; and $18 \times 18 = 324$; Therefore 100 is the transverse axe.

PROPOSITION V.

To find the area of an elliptic segment, cut parallel to either axe.

I. *The transverse and conjugate axes,* (AB, DK) *and the distance* (CF) *of the section, parallel to the conjugate, being known.* Fig. 6.

RULE.

In the circumscribing circle, find the area (*e* B *f*) of the corresponding circular segment, (by Prop. XXII. part I.); multiply this area by the conjugate, the product divided by the transverse, will give the area of the elliptical segment.

EX-

EXAMPLE.

In an ellipsis whose transverse diameter (AB) *is* 120; *its conjugate* (DK) 40: *What is the area of a segment thereof, cut at the distance* (CE) 36, *from the centre.*

Now $\left(\frac{120}{2} - 36 = EB =\right)$ 24 is the height.

And $\left(\frac{4}{3} - \frac{24 \times 32}{60 \times 120 - 24 \times 15} \times \sqrt{120 \times 24} \times 24 =\right)$ 1610,2488 is the area of the circular segment. (art. 132.)

Then $\left(1610{,}2488 \times \frac{40}{120} =\right)$ 536,7496 is the area of the elliptic segment.

II. *The transverse and conjugate axes,* (AB,DK) *and the distance* (CE) *of the section, parallel to the transverse, being known.* Fig. 5.

RULE.

In the inscribed circle find the area *(eDf)* of the corresponding circular segment (by Prop. XXII. part 1.); multiply this area by the transverse; the product divided by the conjugate will give the area of the elliptical segment.

EXAMPLE.

In an ellipsis, whose transverse axe (AB) *is* 120, *its conjugate* (DK) 40: *What is the area of a segment thereof, cut parallel to the transverse, at the distance* (CE) 16 *from the centre?*

Now $\left(\frac{40}{2}-16=ED=\right)$ 4 is the height of the seg.

And $\left(\frac{4}{3}-\frac{4\times32}{80\times40-15\times4}\times\sqrt{40\times4}\times4=\right)65{,}398$ is the area of the circular segment. (art. 132.)

Then $\left(65{,}398\times\frac{120}{40}=\right)$ 196,194 is the area of the elliptical segment.

III. *When there are known, the transverse and conjugate axes;* (AB, DK) *and also the degrees in the circular arc* (*e*D*f*, *e*B*f*,) *of the inscribed or circumscribed circles,*) *cut off by the base* (2EF) *of the segment.* Fig. 4.

RULE.

Multiply the degrees by 0,0174533; from the product take the sine of the given degrees; (found in tables *); multiply the half of the remainder, by half the transverse; the product multiplied by half the conjugate, gives the area required.

* Tables of natural sines; which by this rule are taken as the decimal parts of the radius, supposed to be unity. The most commodious tables are those of *Sherwin.*

EX-

EXAMPLE I.

In an ellipsis, whose transverse axe (AB) *is* 120; *its conjugate* (DK) 40: *Required the area of a segment thereof,* (FDP) *cut parallel to the transverse axe; when the corresponding arc (e* D *f) of the inscribed circle contains* 73° 44′ 24″?

Now 73° 44′ 24″=73,74 degrees,
And its natural sine is 9600015,

Then $\left(\frac{0{,}0174533\times73{,}74-0{,}9600015}{2}\times\frac{120}{2}\times\frac{40}{2}=\right)$
196,2024 is the area.

EXAMPLE II.

In an ellipsis, whose transverse axe (AB) *is* 120; *its conjugate* (DK) 40: *Required the area of a segment thereof,* (FBL) *cut parallel to the conjugate, when the corresponding arc (e* B *f) of the circumscribing circle contains* 106° 15′ 36″?

Now 106° 15′ 36″ = 106,26 degrees,
And its natural sine is 9600015,
Then $\left(\frac{0{,}0174533\times106{,}26-09600015}{2}\times\frac{120}{2}\times\frac{40}{2}=\right)$
536,7536 is the area.

PROPOSITION VI.

To find the solidity of a cylindroid, (or elliptic prism) the diameters of its end, and its length being known.

RULE.

Multiply the area of the end, by the length, and the product will be the solidity required.

EXAMPLE.

What is the solidity of a cylindroid, whose length is 8 feet, and the diameters of either end, are 3 feet and 2 feet?

Then $(3 \times 2 \times 0{,}7854 \times 8 =)$ 37,6992 is the solidity required.

PROPOSITION VII.

To find the convex surface of a cylindroid; the length, and the diameters at either end being known.

RULE.

Find (by Prop. II.) the periphery at either end, and this multiplied by the length will give the convex surface nearly.

EX-

EXAMPLE.

What is the convex ſurface of a cylindroid whoſe length is 8 feet; and the principal diameters of either end, are 3 feet and 2 feet?

Then $\left(\frac{3+2}{2}\times 3{,}1416\times 8=\right)$ 62,832 is the convex ſurface.

Or $\frac{2}{3}=,666\not{6}$; againſt 66 in the tab. p. 259. is 2,6327,
And the diff. $138\times\not{6}=90{,}\dot{8}$ or 91, or 0,0091;
Then $(\overline{2{,}6327+0{,}0091}\times 3\times 8=)$ 63,4032 is the convex ſurface ſought.

PROPOSITION VIII.

To find the ſolidity of a ſpheroid, the axis of rotation, and the revolving axe being known.

RULE.

Multiply the fixed axe by the ſquare of the revolving axe, the product multiplied by 0,5236 $\left(=\frac{p}{6}\right)$ will give the ſolidity.

EXAMPLE I.

What is the ſolidity of a prolate ſpheroid, whoſe tranſverſe axe is 100, and its conjugate is 60?

Then $(100\times\overline{60}^2\times 0{,}5236=)$ 188496 is the ſolidity required.

EXAMPLE II.

What is the ſolidity of an oblate ſpheroid, whoſe longeſt axe is 100, and ſhorteſt axe is 60?

Then $(60\times\overline{100}^2\times 0{,}5236=)$ 314160 is the ſolidity required.

Note, A ſpheroid is $\frac{2}{3}$ of its circumſcribing cylinder.

PROPOSITION IX.

To find the superficial content of a spheroid; the transverse and conjugate axes being known.

RULE.

From the square of half the transverse, take the square of half the conjugate; let the square root of the remainder be called A.

In a prolate spheroid; multiply 0,0174534 by the degrees corresponding to a sine, produced by dividing A by half the fix'd axe: Call the product B.

In an oblate spheroid; multiply 2,302585 by the common logarithm corresponding to the quotient, of the sum of A and half the revolving axe, divided by half the fix'd axe: Call the product B.

Multiply B by the square of half the fix'd axe; divide the product by A; to the quotient add half the revolving axe; the sum multiplied by the revolving axe, and by 3,1416, will give the superficies required.

EXAMPLE I.

What is the superficial content of a prolate spheroid; whose longest diameter is 100, *and the shortest is* 60?

Here 100 is the fix'd axe; and 60 the revolving axe.

Now $\left(\sqrt{\left|\frac{100}{2}\right|^2 - \left|\frac{60}{2}\right|^2} =\right) 40 = A.$

And $(40 \div \frac{100}{2} =)$ 0,8, is the sine of 53,13 degrees nearly.

Then $(53{,}13 \times 0{,}0174533 =) 0{,}927289142 = B.$

Th. $\left(\frac{0{,}927289142 \times \left|\frac{100}{2}\right|^2}{40} + \frac{60}{2} \times 60 \times 3{,}1416 =\right)$ 16579,273 is the superficies sought.

EXAMPLE II.

What is the superficial content of an oblate spheroid, whose longest diameter is 100, *and the shortest is* 60?

Here 60 is the fix'd axe; and 100 the revolving axe.

Now $\left(\sqrt{\left|\frac{100}{2}\right|^2 - \left|\frac{60}{2}\right|^2} =\right) 40 = A.$

And the logarithm of $\left(40 + \frac{100}{2} \div \frac{60}{2} =\right)$ 3, is 0,4771213.

Then $(2{,}302585 \times 0{,}4771213 =) 1{,}0986123 = B.$

Th. $\left(\frac{1{,}0986123 \times \left|\frac{60}{2}\right|^2}{40} + \frac{100}{2} \times 100 \times 3{,}1416 =\right)$ 23473,632 is the superficies sought.

PROPOSITION X.

To find the solidity (ABC$=$S) *of a segment of a spheroid, cut parallel to either axe; those axes,* DE$=r$,FB$=f$) *and the height* (aB$=b$) *of the segment being known.* Pl. III. Fig. 7, 8.

I. *When the section* (AC) *is perpendicular to the axis of rotation* (FB.)

Here the section will be circular.

RULE.

Divide the square of the revolving axe, by the square of the fixed axe; multiply the quotient by thrice the fixed axe, lessened by twice the height of the segment; multiply the product by the square of the said height; this product multiplied by 0,5236 will give the solidity of the segment.

$$\text{Or } S = \frac{rr}{ff} \times \overline{3f - 2b} \times bb \times \frac{p}{6}.$$

EXAMPLE I.

In a prolate spheroid, whose transverse axe is 100, *its conjugate* 60; *what is the solidity of a segment thereof, whose height is* 10, *and cut perpendicular to the transverse axe?*

$$\text{Now } \frac{\overline{60}|^2}{\overline{100}|^2} = 0{,}36.$$

And $100 \times 3 - 10 \times 2 = 280$.

Then ($0{,}36 \times 280 \times \overline{10}|^2 \times 0{,}5236 =$) 5277, 888 is the solidity required.

EX-

EXAMPLE II.

In an oblate spheroid, whose greater diameter is 100, its lesser 60; what is the solidity of a segment thereof, whose height is 12, and cut perpendicular to the conjugate axe?

Now $\frac{\overline{100}|^2}{\overline{60}|^2} = 2{,}\dot{7}$.

And $60 \times 3 - 12 \times 2 = 156$.

Th. $(2{,}\dot{7} \times 156 \times \overline{12}|^2 \times 0{,}5236 =)$ 32672,64 is the solidity required.

II. *When the section* (AC) *is parallel to the axis of rotation* (DE.)

Here the section will be elliptical.

RULE.

Divide the fixed axe by the revolving axe; multiply the quotient by thrice the revolving axe, lessened by twice the height of the segment; multiply the product by the square of the said height; this product multiplied by 0,5236 will give the solidity of the segment.

Or $S = \frac{f}{r} \times \overline{3r - 2h} \times hh \times \frac{p}{6}$.

EXAMPLE I.

In a prolate spheroid, whose transverse axe is 100; *its conjugate* 60; *what is the solidity of a segment thereof, whose height is* 12; *and cut parallel to the transverse axe?*

Now $\frac{100}{60} = 1{,}\dot{6}$.

And $60 \times 3 - 12 \times 2 = 156$.

Then $(1{,}\dot{6} \times 156 \times \overline{12}|^2 \times 0{,}5236 =)$ 19603,584 is the solidity required.

EXAMPLE II.

In an oblate spheroid, whose longest axe is 100; *its shortest axe is* 60; *what is the solidity of a segment thereof, whose height is* 10; *and cut parallel to the conjugate axe?*

Now $\frac{60}{100} = 0{,}6$.

And $100 \times 3 - 10 \times 2 = 280$.

Then $(0{,}6 \times 280 \times \overline{10}|^2 \times 0{,}5236 =)$ 8796,48 is the solidity required.

III. *The solidity of a spheroidical segment may be found by the following*

RULE.

As the solidity of the sphere $\left\{\begin{array}{l}\text{circumscribed,}\\ \text{inscribed.}\end{array}\right.$

To the solidity of the spheroid.

So is the solidity of a spherical segment.

To the solidity of the corresponding elliptical segment.

EX-

EXAMPLES.

I. *In a prolate spheroid, where the longest axe is* 100, *and the shortest is* 60: *What is the solidity of a segment thereof, whose height is* 10, *and cut perpendicular to the transverse axe?*

Now $(\overline{100}|^3 \times 0{,}5236 =)$ 523600 is the solidity of the circumscribing sphere.

And $(\overline{60}|^2 \times 100 \times 0{,}5236 =)$ 188496 is the solidity of the spheroid.

Also $(\overline{100 \times 3 - 10 \times 2} \times \overline{10}|^2 \times 0{,}5236 =)$ 14660,8 is the solidity of the corresponding spherical segment.

Then (523600 : 188496 : : 14660,8 :) 5277, 888 is the solidity of the spheroidal segment required.

II. *In a prolate spheroid whose transverse axe is* 100, *its conjugate* 60; *what is the solidity of a segment thereof, whose height is* 12; *and cut parallel to the transverse?*

Now $\overline{60}|^3 \times 0{,}5236 =)$ 113097,6 is the solidity of the inscribed sphere.

And 188496 is the solidity of the spheroid.

Also $(\overline{60 \times 3 - 12 \times 2} \times \overline{12}|^2 \times 0{,}5236 =)$ 11762,1504 is the solidity of the corresponding spherical segment.

Then (113097,6 : 188496 : : 11762,1504 :) 19603,584 is the solidity of the spheroidal segment required.

III. *In an oblate spheroid, whose longest diameter is* 100, *its shortest* 60; *required the solidity of a segment whose height is* 12; *and cut parallel to the longer axe?*

Now the solidity of the inscribed sphere is 113097,6.

And the solidity of the spheroid is 314160.

And the solidity of the corresponding spherical segment is 11762,1504.

Then (113097,6 : 314160 : : 11762,1504 :) 32672,64 is the solidity of the spheroidal segment.

IV. *In an oblate spheroid, whose principal axes are* 100 *and* 60; *required the solidity of a segment whose height is* 10; *and cut parallel to the shortest axe?*

Now the solidity of the circumscribing sphere is 523600.

And the solidity of the spheroid is 314160.

Also the solidity of the corresponding spherical segment is 14460,8.

Then (523600 : 314160 : : 14660,8 :) 8796,48 is the solidity required.

PRO-

PROPOSITION XI.

To find the solidity (S) *of a frustum* (DEAC) *of a spheroid, one end* (DE) *passing thro' the centre* (G) *of the spheroid, perpendicular to an axe* (FB); *the diameters* (DE=D, AC=d) *of the ends, and their distance*(Ga=b)*being known.* Pl. III. Fig. 7, 8.

I. *When the ends are perpendicular to the axis of rotation.*

Here each end will be circular.

RULE.

To twice the square of the diameter of the greater end, add the square of the diameter of the lesser end; multiply the sum by the distance of the ends; the product multiplied by 0,2618 will give the solidity required.

$$\text{Or } S = \overline{2\,DD + dd} \times b \times \frac{p}{12}.$$

Ex. I. *What is the solidity of a frustum of a prolate spheroid, the ends being perpendicular to the transverse axe; the diameter of the greater end being* 60, *that of the lesser end* 36; *and the distance of the ends* 40?

Then ($\overline{\overline{60}^2 \times 2 + \overline{36}^2} \times 40 \times 0{,}2618 =$)88970, 112 is the solidity sought.

Ex. II.

Ex. II. *What is the solidity of the frustum of an oblate spheroid, the ends being perpendicular to the conjugate axe; the diameter of the greater end being* 100, *that of the lesser end* 80, *and the distance of the ends* 18?

Th. ($\overline{\overline{100}^2 \times 2 + \overline{80}^2} \times 18 \times 0,2618 =$) 124407, 36 is the solidity sought.

II. *When the ends are parallel to the axis of rotation.*

Here each end will be elliptical.

RULE.

Multiply twice the transverse diameter of the greater end, by its conjugate; to the product, add the rectangle under the transverse and conjugate diameters of the lesser end; multiply the sum by the distance of ends, the product multiplied by 0,2618 will give the solidity required.

Or putting T,C, for the longest and shortest diameters of the greater end.

t,*c*, those of the lesser end.

Then $S = \overline{2\,TC + tc} \times h \times \frac{p}{12}$.

Ex. III. *In the frustum of a prolate spheroid, the ends being parallel to the transverse axe; the two diameters of the greater end, are* 100; 60; *and the two diameters of the lesser end, are* 80; 48; *the distance of the ends is* 18: *Required the solidity of that frustum?*

Then ($\overline{2 \times 100 \times 60 + 80 \times 48} \times 18 \times 0,2618 =$) 74644,416 is the solidity sought.

Ex-

Ex. IV. *In the frustum of an oblate spheroid, the ends being parallel to the conjugate axe; the two diameters of the greater end are* 100; 60; *the two diameters of the lesser end are* 60; 36; *and the distance of the ends is* 40: *Required the solidity of that frustum?*

Then $(\overline{2\times100\times60+36\times60}\times40\times0{,}2618=)$ 148283,52 is the solidity sought.

PROPOSITION XII.

To find the superficial content of a frustum of a spheroid, one of whose parallel ends, passes thro' the centre of the spheroid, perpendicular to the fix'd axe, the diameters of the ends of the frustum, and their distance being known.

RULE.

1. Find the fix'd axe;

2. Let the square root, of the difference of the squares, of half the fix'd, and half the revolving axe, be called A.

3. Multiply the square of A, by the square of the length; $\left\{\begin{matrix}\text{take}\\ \text{add}\end{matrix}\right\}$ the product $\left\{\begin{matrix}\text{from}\\ \text{to}\end{matrix}\right\}$ the fourth power of half the fix'd axe; let the square root of the $\left\{\begin{matrix}\text{remainder}\\ \text{sum}\end{matrix}\right\}$ be called B.

4. In

4. In a prolate ſpheroid; multiply A, by the length of the fruſtum; divide the product by the ſquare of half the longeſt axe; ſeek the quotient among the ſines; multiply the correſponding degrees by 0,0174534: Call the product D.

4. In an oblate ſpheroid; multiply A by the length; to the product add B; divide the ſum by the ſquare of half the ſhorteſt axe; multiply the logarithm of the quotient by 2,302585: Call the product D.

5. Divide the ſquare of half the fix'd axe by A; multiply the quotient by D: Alſo, divide the length by the ſquare of half the fix'd axe; multiply the quotient by B: The ſum of the two products, multiplied by half the revolving axe, and by 3,1416, will give the ſuperficies required.

Note, The difference between the convex ſurface of the hemiſpheroid and this fruſtum, will give the convex ſurface of a ſegment of a ſpheroid.

EX-

EXAMPLES.

I. *What is the convex surface of a frustum of a prolate spheroid, the diameter of the lesser end being* 36; *that of the greater end, or revolving axe,* 60; *and the distance* 40?

Now $\sqrt{\overline{\frac{60}{2}}\Big|^2 - \overline{\frac{36}{2}}\Big|^2} = 24$

And $\frac{\frac{1}{2}60 \times 40}{24} \times 2 = 100$ is the fix'd axe.

And $\sqrt{\overline{\frac{100}{2}}\Big|^2 - \overline{\frac{60}{2}}\Big|^2} = 40 = A$.

Also $\frac{40 \times 40}{50 \times 50} = 0{,}64$ is the sine of 39,79 degrees.

Then $(39{,}79 \times 0{,}0174534 =)\ 0{,}69447 = D$.

And $(\sqrt{\overline{50}|^4 - \overline{40}|^2 \times \overline{40}|^2} =)\ 1920{,}94 = B$.

Then $\left(\frac{50 \times 50}{40} \times 0{,}69447 =\right)\ 43{,}404375 =$ one product.

And $\frac{40}{50 \times 50} \times 1920{,}94 = 30{,}7349968 =$ other product.

Theref. $43{,}404375 + 30{,}7349968 = 74{,}139372$.

Then $(74{,}139372 \times \frac{60}{2} \times 3{,}1416 =)\ 6987{,}518947$ is the surface required.

II. *What is the convex surface of a frustum of an oblate spheroid, the diameter of the lesser end being* 80, *that of the greater end, or revolving axe,* 100; *and the distance of the ends* 18?

Now $\sqrt{\left|\overline{\frac{100}{2}}\right|^2 - \left|\overline{\frac{80}{2}}\right|^2} = 30.$

And $\left(\frac{100 \times 18}{30} \times 2 =\right)$ 60 is the fix'd axe.

Then $\left(\sqrt{\left|\overline{\frac{100}{2}}\right|^2 - \left|\overline{\frac{60}{2}}\right|^2} =\right) 40 = A.$

And $\left(\sqrt{\overline{30}|^4 + \overline{40}| \times \overline{18}|^2} =\right) 1152{,}5624 = B.$

Also $\frac{40 \times 18 + 1152{,}5264}{30 \times 30} = 2{,}080625$; whose log. is 0,3181938.

Theref. $(0{,}3181938 \times 2{,}302585 =) 0{,}7326682 = D.$

Now $\left(\frac{30 \times 30}{40} \times 0{,}7326682 =\right) 16{,}4850345 =$ one product.

And $\frac{18}{30 \times 30} \times 1152{,}5624 = 23{,}051248 =$ other product.

Also $16{,}4850345 + 23{,}051248 = 39{,}5362825.$

Then $(39{,}5362825 \times \frac{100}{2} \times 3{,}1416 =) 6210{,}359254$ is the surface required.

PRO-

PROPOSITION XIII.

In an elliptical ſpindle (KDI.A) *the length or axes* (KL $=l$) ; *a perpendicular diameter* (DA $=2$D) *in the middle* ; *and another parallel thereto* (EH $=2d$) *biſecting the half length, being known* ; *to find the axes* (FB $=2t$, GD $=2c$) *of the generating ellipſis.* Pl. III. Fig. 9.

RULE.

From the ſquare of half the greater diameter, take the ſquare of the difference of the two diameters; divide the remainder by half the greater diameter, leſſened by twice the difference of the two diameters; the quotient will be that diameter of the ellipſis, to which the axe of the ſpindle is perpendicular.

Or $\dfrac{DD-\overline{D-d}^2}{D-2\overline{D-2d}}$ = an axe of the ellipſe.

And the tranſverſe will be found by caſe III. prop. I.

Ex. *In an elliptical ſpindle, whoſe length is* 80 ; *the greater diameter* 24 ; *and the diameter at a quarter the length is* 18,99094 : *Required the tranſverſe and conjugate axes of the generating ellipſe?*

Now $24 - 18,99094 = 5,00906$ the difference of the two diameters,

Then $\left(\dfrac{\overline{12}^2 - \overline{5,00906}^2}{12-10,01812} = 59,99,\ \text{\&c. or}\right)$ 60 is the conjugate axe.

Hence the tranſverſe will be 100.

In

In the frustum of an elliptical spindle, where one end passes thro' the centre of the spindle; the length of the frustum; the diameters of the ends; and a diameter equally distant from the ends, being known; the axes of the ellipse will be found as follows.

RULE. From the square of half the difference, of the greater and lesser diameters, take the square of the difference of the greater and mean diameters for a dividend. From half the difference of the greater and lesser diameters, take twice the difference of the greater and mean diameters for a divisor; the quotient will be that axe of the ellipse, to which the section is perpendicular.

Ex. *In the frustum of an elliptical spindle (parallel to the transverse) whose length is* 14; *the diameter of the greater end* 24; *that of the lesser end* 21,6; *and the diameter in the midway* 23,40909: *What are the axes of the ellipse?*

Now $\left(\frac{24}{2} - \frac{21,6}{2} = \right)$ 1,2 is the half difference of the greater and lesser diameters.

And (24—23,40909=) 0,59091 is the difference between the greater and mean diameters.

Then $\left(\frac{\overline{1,2}^2 - \overline{0,59091}|^2}{1,2 - 0,59091 \times 2} = \right)$ 60 is the conjugate axe.

And 100 is the transverse axe.

PRO-

PROPOSITION XIV.

To find the solidity (S) *of an elliptical spindle* (KDLA)*; wherein the length of its axe* (KL$=l$)*; its greatest diameter* (AD$=$D)*: and the diameter* (EH) *at a quarter of the length from one end are known; the axe of the spindle being parallel to that of the generating ellipsis.* Pl. III. Fig. 9.

RULE.

1. Find the axes of the ellipse, and the central distance (GI$=a$).

2. Find the area (A) of an elliptic segment, the measures of whose base and height, are respectively equal to those of the axe and half diameter of the spindle.

3. Divide this area, by a third of the spindle's length; subtract the quotient from the spindle's diameter; multiply the remainder by eight times the central distance; to the product add twice the square of the given diameter; multiply the sum by the given length; the product multiplied by 0,2618 will give the solidity required.

Or $S = D - \frac{A}{\frac{1}{3}l} \times 8a + 2DD \times l \times \frac{p}{12}$.

EX-

EXAMPLE.

What is the ſolidity of an elliptical ſpindle, the length of whoſe axe is 80; *the greateſt diameter is* 24; *and the diameter at* 20, *from the end, is* 18,99094?

Now the conjugate and tranſverſe diameters of the ellipſe, are 60 and 100.

Then $\left(\frac{60}{2}-\frac{24}{2}=\right)$ 18 is the diſtance of the centres.

Alſo the area of the elliptic ſegment, whoſe baſe is 80, and height 12; will be found equal to 670,942.

Then $\frac{670,942}{80 \div 3} = 25,160325$

And $24 - 25,160325 = -1,160325$

Therefore $-1,160325 \times 8 \times 18 = -167,0868$.

Then $(\overline{2 \times \overline{24}|^2 - 167,0868} \times 80 \times 0,2618 =)$ 20628,022 is the ſolidity ſought.

PRO-

PROPOSITION XV.

To find the solidity (S) *of a frustum* (DAHE) *of an elliptical spindle, one of whose parallel ends* (AD) *passes thro' the centre* (I) *of the spindle: The diameters* (AD=D,EH=d), *of those ends, their distance* (IB=l), *and a diameter* (ef) *taken in the midway, being known.* Pl. III. Fig. 9.

RULE.

Find the axes of the ellipse and the central distance (GI=a).

Find the area (A) of an elliptical segment, whose base is twice the length of the frustum; and whose height is equal to the difference of half the diameters of the ends.

Divide half this area by a third of the length; to the quotient add the lesser diameter; subtract the sum from the greater diameter; multiply the remainder by eight times the central distance; to the product add the sum of the square of the lesser diameter, and twice the square of the greater diameter; multiply the sum by the given length; the product multiplied by 0,2618 will give the solidity required.

$$\text{Or } S = D - \frac{\frac{1}{2}A}{\frac{1}{3}l} + d \times 8a + 2DD + dd \times l \times \frac{p}{12}.$$

EX-

EXAMPLE.

What is the solidity of a frustum of an elliptical spindle, whose length is 14; the diameter at the greater end is 24; that at the lesser end is 21,6; and a diameter in the mid-way is 23,40909?

Now 60 and 100 will be found for the conjugate and transverse axes of the ellipse.

And $\left(\frac{60}{2}-\frac{24}{2}=\right)$ 18 is the central distance.

Also 22,48937 is the area of an elliptical segment, whose height is 1,2 and base 28.

Its half is 11,24468.

Then $\frac{11,24468}{14 \div 3}+21,6=24,00957$.

And $\overline{24-24,00957}\times 8\times 18=-1,37808$.

Also $2\times\overline{24}|^2+\overline{21,6}|^2=1618,56$.

Then $(\overline{1618,56-1,37808}\times 14\times 0,2618=)$ 5927,29515 is the solidity required.

SEC-

SECTION II.

Of parabolic lines, superficies, and solids.

PROPOSITION XVI.

To find the area of a parabola, the double ordinate (or base CD), *and axis (or height* AB) *being known.* Pl. III. Fig. 10.

RULE.

Multiply the base by the height; and two thirds of this product will be the area required.

EXAMPLE.

What is the area of a parabola, whose axis is 12, *and the double ordinate is* 16?

Th. ($16 \times 12 \times \frac{2}{3} =$) 128 is the area.

Note, Every conical parabola is $\frac{2}{3}$ of its circumscribing parallelogram.

PROPOSITION XVII.

To find the area (A) *of a frustum* (DHGC) *of a parabola, whose parallel ends* (DC=B, GH=b), *and their distance* (IB=d) *are known.* Pl. III. Fig. 10.

O RULE.

RULE.

To the ſquare of the greater end, add the ſquare of the leſſer end, and the product of the ends; divide the ſum, by the ſum of the ends; the quotient multiplied by two thirds of the diſtance of the ends will give the area ſought.

$$\text{Or } A = \frac{BB + bb + Bb}{B + b} \times \tfrac{2}{3}d.$$

EXAMPLE.

Suppoſe the end CD *is* 24; *the end* GH *is* 20; *and the diſtance* IB, $5\frac{1}{2}$: *Required the area* CGHD?

$$\text{Then } \left(\frac{\overline{24}|^2 + \overline{20}|^2 + 24 \times 20}{24 + 20} \times 5{,}5 \times \tfrac{2}{3} =\right) 121{,}3$$

is the area required.

PROPOSITION XVIII.

To find the length (L) *of the curve of a parabola, whoſe abſciſs* ($AB = x$), *and ordinate* ($CB = y$) *are known.* Pl. III. Fig. 10.

RULE.

Divide the ſquare of the ordinate by twice the abſciſs, the quotient is the half parameter.

$$\text{Or } p = \frac{yy}{2x}.$$

To

To the square of the half parameter add the square of the ordinate, the square root of the sum, call A.

Or $A = \sqrt{pp + yy}$.

Multiply A by the ordinate, divide the sum by half the parameter; call the quotient B.

Or $B = \frac{yA}{p}$.

Add A to the ordinate, divide the sum by half the parameter, seek the tabular logarithm to the quotient; multiply the logarithm by 2,302585; the product multiplied by half the parameter, and B added to the product, gives the length of the curve required.

Or $L = B + 2{,}302585 \times \log. \frac{A + y}{p}$.

EXAMPLE.

What is the length of the curve of a parabola, whose abscis is 50, *and ordinate* 30?

Now $\left(\frac{\overline{30}|^2}{50 \times 2} =\right)$ 9 is the half parameter.

And $\left(\sqrt{\overline{9}|^2 + \overline{30}|^2} =\right) 31{,}321 = A$.

Also $\left(\frac{31{,}321 \times 30}{9} =\right) 104{,}403 = B$.

But $\frac{31{,}321 + 30}{9} = 6{,}8134$.

Its logarithm $= 0{,}8333639$.

Then $(\overline{0{,}8333639 \times 2{,}302585 \times 9 + 104{,}403333}$ $=)$ 121,673352 is the length of the curve required.

PROPOSITION XIX.

To find the solidity of a parabolic conoid (ABD, A*af*); *the diameter* (BD, *or fa, mn*) *of the base, and the height* (AC *or rs*) *being known.* Pl. III. Fig. 11.

RULE. Multiply the square of the diameter of the base, by the height; then the product multiplied by 0,3927 will give the solidity.

Or, multiply the area of the base by half the height, and the product will be the solidity.

Note, A paraboloid is half of its circumscribing cylinder.

EXAMPLES.

I. *What is the solidity of a paraboloid, whose height is* 50; *and the diameter of its circular base is* 60?

Then ($\overline{60}|^2 \times 50 \times 0,3927 =$)70686 is the solidity required.

II. *In the segment of a paraboloid, the greater and lesser diameters of the elliptic base are* 300 *and* 60; *and the height is* 10: *Required the solidity of that segment?*

Then ($300 \times 60 \times 0,7854 \times 10 =$) 70686 is the solidity sought.

PRO-

PROPOSITION XX.

To find the convex superficies (s) of a parabolic conoid (ABD); *the abscis* (AC $=x$) *and ordinate* (BC $=$ CD $=y$) *of the generating parabola being known.* Pl. III. Fig. 11.

RULE.

To four times the square of the abscis, add the square of the ordinate; let the square root of the sum be called A. Or A $= \sqrt{4xx+yy}$.
To A, add the ordinate; let their sum be a divisor to the square of the ordinate; to the quotient add A; multiply the sum by the ordinate; the product multiplied by 2,0944, will give the convex superficies required.

$$\text{Or } s = \overline{A + \frac{yy}{A+y}} \times y \times \tfrac{2}{3} p.$$

EXAMPLE.

What is the convex superficies of a paraboloid, the diameter of whose base is 60, and the height 50? *Or the ordinate* 30, *and abscis* 50?

Now $(\sqrt{4 \times 50 \times 50 + 30 \times 30} =)$ 104,403 $=$ A.

And $\left(\frac{900}{104{,}403 + 30} =\right)$ 6,69628 is the quotient.
Also 6,69628 $+$ 104,403 $=$ 111,09928.
Then (111,09928 $\times$ 30 $\times$ 2,0944 $=$) 6980,58996 is the convex superficies required.

PROPOSITION XXI.

To find the solidity (S) *of a frustum* (BD*db*) *of a paraboloid, contained between two parallel planes, each perpendicular to the axe* (AC); *the diameters* (BD=D, *bd*=*d*) *at those sections, and the distance* (C*c*=*h*) *of the ends being known.* Pl. III. Fig. 12.

RULE.

To the square of the diameter of the greater end, add the square of the diameter of the lesser end; multiply the sum by the length, or height; and the product multiplied by 0,3927 will give the solidity.

Or $S = \overline{DD + dd} \times h \times {}'p$.

EXAMPLE.

What is the solidity of a parabolic frustum, the diameter of the greater end being 60, *that of the lesser end* 48, *and the distance of the ends being* 18?

Then $(\overline{\overline{60}^2 + \overline{48}^2} \times 18 \times 0{,}3927 =)$ 41733,0144 is the solidity required.

PROPOSITION XXII.

In a paraboloid (ABD) *whose base* (B*b*D*d*) *is perpendicular to the axe* (AC). *To find the solidity* (S) *of a segment thereof,* (*a*D*bd*) *cut parallel to the axe; the abscis* (*ac*=*h*) *the ordinate* (*dc*=*cb*=*y*) *at the section, and also the height* (D*c*=*x*) *of the circular segment of its end being known.* Pl. III. Fig. 11.

RULE.

RULE.

Find the radius (r) of the base of the paraboloid; divide the square of this radius by the square of the ordinate; multiply the quotient by half the circular segment, ($b\mathrm{D}d = 2\mathrm{A}$) at the end of the conoidal segment; (found prop. XXII. part 1.) from the product subtract one third of the ordinate, multiplied by the distance thereof from the axe; the remainder multiplied by the abscis, or length of the conoidal segment, will give the solidity required.

$$\text{Or } S = \overline{\frac{rr}{yy} \times A - \frac{1}{3} y \times \overline{r - x}} \times h.$$

EXAMPLE.

In a slice (aDbd) cut from a paraboloid parallel to the axe, and perpendicular to the base thereof, the abscis (ac) is 18; *the ordinate (dc = cb) is* 18; *and the height (Dc) of the circular segment at the end is* 6: *Required the solidity of the slice, or conoidal segment?*

Now $\left(\frac{18 \times 18 + 6 \times 6}{2 \times 6} = \right)$ 30 is the radius.

And 147,35298 is the area of the circular segment $b\mathrm{D}d$; its half is 73,67649.

Then $\overline{\frac{30 \times 30}{18 \times 18} \times 73{,}67649 - \frac{18 \times \overline{30 - 6}}{3}} \times 18 =)$ 1091,824488 is the solidity of the segment.

PROPOSITION XXIII.

To find the solidity (S) *of a parabolic spindle* (BACD), *the axis of rotation* ($BC=l$), *and the greatest diameter* ($AD=D$) *of the solid being known.* Pl. III. Fig. 13.

RULE.

Multiply the square of the diameter by the length, the product multiplied by 0,418879 will give the solidity.

Or $S = DD \times l \times \frac{2}{15} p$.

EXAMPLE.

Suppose the length of a parabolic spindle be 9 *feet, and the greatest diameter is* 3 *feet: Required the solidity?*

Then ($3 \times 3 \times 9 \times 0{,}418879 =$) 33,929199 is the solidity required.

Note, A parabolic spindle is $\frac{2}{15}$ of its circumscribing cylinder.

PRO-

PROPOSITION XXIV.

To find the ſolidity (S) *of a fruſtum, or zone,* (AFED) *of a parabolic ſpindle, contained between two parallel ends* (AD,FE) *the greater wherof* (AD) *paſſes through the middle* (I) *of the ſpindle, perpendicular to the axe* (BC). *The diameters* ($AD = D, FE = d$) *of the ends, and their diſtance* ($IG = l$) *being known.*

RULE.

To eight times the ſquare of the greater diameter, add thrice the ſquare of the leſſer diameter, and four times the product of the two diameters; multiply the ſum by the diſtance of the ends, the product multiplied by 0,05236 will give the ſolidity.

$$\text{Or } S = \overline{8DD + 3dd + 4Dd} \times l \times \frac{p}{6}.$$

EXAMPLE.

What is the ſolidity of a fruſtum, or zone of a parabolic ſpindle, the diameter of the greater end being 36 *inches, that of the leſſer end* 20 *inches, and the diſtance of the ends* 36 *inches?*

Now $36 \times 36 \times 8 = 1036$
And $20 \times 20 \times 3 = 1200$
Alſo $36 \times 20 \times 4 = 2880$
Their ſum is 14448

Then $14448 \times 36 \times 0,05236 = 27233,9$ is the ſolidity required.

SECTION III.

Of hyperbolic lines, superficies and solids.

In elliptic and parabolic figures, the lines contained or drawn within them, are sufficient to determine most of the problems that can be proposed concerning their superficies, solidities, &c. But such things cannot so conveniently be done in the hyperbola, without knowing the relation which some lines within the figure have to others that lie without it, and of these the most useful are the transverse and conjugate diameters. In Pl. III. Fig. 14.

The curves GAH, IBK, are hyperbolas; where GH, IK, are double ordinates, AF, BL, are abscissas; AB is the transverse axe, DE its conjugate; the lines C*a*, drawn thro' the centre C, are called assymptotes.

PROPOSITION XXV.

To find the transverse and conjugate axes ($AB = tt$, $DE = 2c$) *of an hyperbola, whose abscis* ($AF = x$), *and ordinate* ($GF = y$), *are known; and also the length of the abscis* ($Af = z$) *corresponding to an ordinate* ($gf = v$) *equal to half the given one.* (*viz.* $gf = \frac{1}{2}GF$). Pl. III. Fig. 14.

RULE I.

From the square of the greater abscis, take 4 times the square of the lesser abscis; divide the remainder by four times the lesser abscis lessened by the greater abscis; the quotient will shew the length of the transverse sought.

Or $2t = \dfrac{xx - 4zz}{4z - x}$.

EX-

EXAMPLE.

What is the transverse axe of an hyperbola, whose abscis is 40, ordinate 48; and the abscis corresponding to an ordinate of 24, is 12,111?

Then $\left(\frac{\overline{40}^2 - 4 \times \overline{12,111}^2}{4 \times \overline{12,111 - 40}} = \right)$ 120 is the transverse axe required.

RULE II.

Multiply the abscis by the sum of the transverse and abscis; let the square root of the product be a divisor to the product of the transverse and ordinate; the quotient will be the conjugate required.

Or $2c = \frac{2ty}{\sqrt{x \times \overline{2t + x}}}$

EXAMPLE.

Where the transverse is 120, the abscis 40, and the ordinate 48: What is the conjugate?

Now $(\sqrt{\overline{120 + 40} \times 40} =)$ 80 is the divisor.

Then $\left(\frac{120 \times 48}{80} = \right)$ 72 is the conjugate sought.

PROPOSITION XXVI.

To find the transverse axe ($=2t$) *of an hyperbola, wherein are known, three equidistant ordinates*; (GF$=y$, *be*$=s$, *gf*$=v$), *and also, their distance* (F*e*$=$*ef*$=d$) *from each other.* Plate III. Fig. 14.

RULE.

1. Let the difference of the squares of the mean and lesser ordinates, be called B.

Or put $B = ss - vv$.

2. And the difference of the squares of the greater and lesser ordinates, be called D.

Or put $D = yy - vv$.

3. From four times B, take D, for a dividend; and from D, take twice B for a divisor: Let the quotient be called A.

Or put $A = \frac{4B - D}{D - 2B}$.

4. To A, add 1; multiply the sum by the square of the lesser ordinate; divide the product by B; subtract the quotient from the square of half A; the square root of the remainder multiplied by the common distance of the ordinates, will give half the transverse axe.

Or $t = d\sqrt{\overline{\tfrac{1}{2}A}^2 - \frac{\overline{A+1} \times vv}{B}}$.

EX-

EXAMPLE.

Let the greater ordinate be 48; *the lesser* 27; *the mean ordinate* 38,2132196; *and their common distance* 12,5: *Required the transverse axe of this hyperbola?*

Now $(\overline{38{,}2132196}|^2 - \overline{27}|^2 =)$ $731{,}250152 = B$.

And $(\overline{48}|^2 - \overline{27}|^2 =)$ $1575 = D$.

Also $\left(\dfrac{4 \times 731{,}250152 - 1575}{1575 - 2 \times 731{,}250152} =\right) 12 = A.$

Then $\dfrac{\overline{12 + 1} \times \overline{27}|^2}{731{,}250152} = 12{,}96.$

And $\left(\sqrt{\overline{\left|\dfrac{12}{2}\right|^2 - 12{,}96 \times 12{,}5}} =\right)$ 60 is the half transverse sought.

PROPOSITION XXVII.

In an hyperbola, whose transverse axe $(=t)$ *conjugate axe* $(=c)$, *and abscissa* $(=x)$, *being known; to find the area.*

RULE.

RULE.

To the transverse add $\frac{5}{7}$ of the abscis, multiply the sum by the abscis, and take 21 times the square root of the product; call this A.

Or put $A = 21\sqrt{\overline{t + \frac{5}{7}x} \times x}$.
To A add four times the square root of the product of the transverse and abscis; call this B.

Or put $B = A + 4\sqrt{tx}$.

Divide the conjugate by the transverse; multiply the quotient by $\frac{4}{75}$ of the abscis; the product multiplied by B gives the area.

Or area $= \frac{c}{t} \times \frac{4}{75}x \times B$.

Ex. *Suppose the transverse is* 100, *the conjugate* 60, *and the abscis* 50: *Required the area of the hyperbola?*

Now $(21 \times \sqrt{\overline{100 + \frac{5}{7} \times 50} \times 50} =) 1729{,}88445$ $= A$.

And $(1729{,}88445 + 4 \times \sqrt{100 \times 50} =) 2012{,}72717$ $= B$.

Then $\left(\frac{60}{100} \times \frac{4 \times 50}{75} \times 2012{,}72717 =\right) 3220{,}363472$ is the area required.

PRO-

PROPOSITION XXVIII.

To find the solidity (S) of a hyperbolic conoid, the height, (or length $=x$) the diameter of the base (or end $=$ D) and the transverse diameter ($=t$) of the generating hyperbola being known.

RULE.

To thrice the transverse, add twice the height; divide the sum by the sum of the transverse and height; multiply the quotient by the height; the product multiplied by the square of the diameter, and by 0,1309, will give the solidity.

$$\text{Or } S = \frac{3t + 2x}{t + x} \times x \times DD \times \frac{p}{24}.$$

EXAMPLE.

What is the solidity of a hyperboloid, whose height (or length) is 50; *the diameter of the base (or end) is* 103,923048; *and the transverse axe is* 100?

Now $\left(\frac{100 \times 3 + 50 \times 2}{100 + 50} = \frac{8}{3} = \right)$ 2,6̸ is the quotient.

And $(\overline{103{,}923048}|^2 =)$ 10799,99977, or 10800 is the square of the diameter.

Then (2,6̸ × 50 × 10800 × 0,1309 =) 188496 is the solidity required.

PRO-

PROPOSITION XXIX.

To find the convex surface (*s*) *of a hyperboloid, the diameter* (D) *of whose base and the height* (*h*) *are known: And also, the distance from the vertex, where the diameter is equal to half that of the base.*

RULE.

1. Find the transverse ($=2t$) and conjugate ($=2c$) axes.

2. Let the square root, of the sum of the squares, of the half transverse and half conjugate be called A. Or put $A = \sqrt{tt + cc}$.

3. Multiply the square of the sum of the height and half transverse, by the square of A; from the product take the fourth power of the half transverse; let the square root of the remainder be called B.

Or put $B = \sqrt{AA \times \overline{t + x}^2 - t^4}$

4. Multiply the sum of A, and the half conjugate, by the half transverse, for a dividend: Multiply A by the sum of the height and half transverse; let the product added to B be a divisor: Multiply the log. of the quotient by 2,302585; call the product C.

$$\text{Or put } C = 2{,}302585 \times \log. \frac{\overline{A + c} \times t}{B + \overline{t + x} \times A}.$$

5. Divide the square of the half transverse by A, multiply the quotient by C; divide the sum of the height and half transverse by the square of the half transverse, multiply the quotient by B: From the sum of the two products take the half conjugate; the remainder

remainder multiplied by the half conjugate, and the product by 3,1416, will give the convex surface required.

$$\text{Or } s = \overline{\frac{tt}{A} \times C + \frac{t+x}{tt} \times B - c \times c} \times p.$$

EXAMPLE.

What is the superficial content of a hyperboloid; the diameter of whose base is 48; *the height* 40; *and the distance of the vertex, from the place of the diameter,* 24; *is* 12,111?

By (pr. 25.) the transverse is 120; and the conjugate is 72.

$$\text{Now } \left(\sqrt{\overline{\tfrac{120}{2}}\Big|^2 + \overline{\tfrac{72}{2}}\Big|^2} =\right) 69,97142 = A.$$

$$\text{And} \left(\sqrt{\overline{60+40}|^2 \times \overline{69,97142}|^2 - \overline{60}|^4} =\right) 6000 = B.$$

$$\text{Also} \left(\frac{\overline{69,97142+36} \times 60}{\overline{40+60} \times 69,97142+6000} =\right) 0,4892064;$$

whose log. is $\bar{1}$,6894922.

Then ($\bar{1}$,6894922 × 2,302585 =) $\bar{1}$,2850296 = C = − 0,7149704.

$$\text{Again, } \left(\frac{\overline{36}^2}{69,97142} \times -0,7149709 = -36,77473;\right.$$

$$\text{And } \frac{40+60}{60 \times 60} \times 6000 = 166,\not{6}.$$

Now, 166,$\not{6}$ − 36,77473 − 36 = 93,89193.

Then 93,89193 × 36 × 3,1416 = 10618,9518 is the convex surface sought.

PRO-

PROPOSITION XXX.

To find the ſolidity (S) *of a fruſtum of a hyperbolic conoid; the diameters of the ends,* (*viz.* 2 D = *greater* 2*d* = *leſſer*) *and their diſtance* (= *h*) *being known; and alſo, a diameter taken in the midway between the ends.*

R U L E.

Find the tranſverſe axe (= 2t) and the conjugate (= 2c.)

Divide the ſquare of the half conjugate by the ſquare of the half tranſverſe; multiply the quotient by a third of the ſquare of the diſtance of the ends; ſubtract the product from the ſum of the ſquares of the half diameters of the ends; the remainder multiplied by the diſtance of the ends, and the product by 1,5708, will give the ſolidity of the fruſtum required.

$$\text{Or } S = \overline{DD + dd - \frac{cc}{tt} \times \frac{1}{3} hh} \times h \frac{p}{2}.$$

EX-

EXAMPLE.

What is the solidity of a frustum of a hyperbolic conoid, whose greater diameter is 96; lesser diameter 54; middle diameter is 76,4264392; and the common distance of these diameters is 12,5?

Now 60 is the half transverse,
And 36 is the half conjugate,

Then $\frac{\overline{36}|^2}{\overline{60}|^2} \times \frac{\overline{25}|^2}{3} = 75.$

And $\left(\overline{\frac{96}{2}}\right|^2 + \overline{\frac{54}{2}}\Big|^2 = \Big)$ 3033 is the sum of the squares of the half ends.

Then $(\overline{3033 - 75} \times 25 \times 1{,}5708 =)$ 116160,66 is the solidity sought.

SCHOLIUM.

The superficial contents of some of the solids in the three foregoing sections are omitted, because they did not appear to be reducible to easy practical rules: Beside, a multitude of other problems might have been added, concerning the solids that can be produced by the rotation of the conic sections about their axes, abscisses, ordinates, tangents, and assymptotes: But as such problems (and indeed several in this work) seem to be of little more use than the exercise of the elements, by which they may be computed; therefore 'tis more proper to seek for them among the treatises of exhaustions, indivisibles, infinites, and fluxions; particularly the latter, which is most in use; being far more extensive than either of the former, and best suited to difficult enquiries in mathematical subjects.

SEC-

SECTION IV.

Of the solidity and superficies of cylindric rings.

DEFINITION.

If a cylinder be circularly bent, until its ends meet, the figure thus formed may be called a cylindric ring.

Or this solid may be conceived to be generated by the rotation of a circle, about a right line, as an axe, either touching the circle, or at a given distance from it.

A great variety of solids may be conceived to be thus generated from different planes, such as elliptic, parabolic, hyperbolic, *&c.* but in this work, no other will be considered but that which arises from a circle.

By thickness is to be understood the diameter of the generating circle.

The inner diameter is twice the distance of the axe from the generating circle.

PRO-

PROPOSITION XXXI.

To find the ſolidity of a cylindric ring, whoſe thickneſs, and inner diameter, are known.

RULE.

To the thickneſs of the ring add the inner diameter; multiply the ſum by the ſquare of the half thickneſs; the product multiplied by 9,8696044 $(=pp)$ will give the ſolidity ſought.

EXAMPLE.

What is the ſolidity of an anchor ring, whoſe inner diameter is 8 inches, and thickneſs in metal 3 inches?

Then $(\overline{8+3} \times \overline{\tfrac{3}{2}}|^2 \times 9{,}8696044 =)$ 488,5454 is the ſolidity of that ring.

When the inner diameter is nothing; this rule will alſo give the ſolidity.

PRO-

PROPOSITION XXXII.

To find the convex superficies of a cylindric ring, whose thickness and inner diameter are known.

RULE.

To the thickness of the ring add the inner diameter; multiply the sum by the thickness; the product multiplied by 9,8696044, will give the superficies required.

EXAMPLE.

A jeweller would have for his sign a cylindric ring, whose outside diameter shall be 18 *inches, and* 3 *inches thick: What will the gilding of this ring come to at a penny an inch?*

Now $18 - 3 \times 2 = 12 =$ inner diameter.

Then $(\overline{3 + 12} \times 3 \times 9{,}8696044 =)\ 444{,}132$ inches is the convex surface.

Therefore the expence will be 1 £. 17 *s.*

SECTION V.

Of arched roofs.

When a building is covered with an arched roof, such roofs are, either, Vaults, Domes, Salons, *or* Groins.

VAULTS, When the curved sides of the roof spring from opposite or side walls, and meet in a right line over the middle of the building.

Such are the middle Isles of most churches.

DOME, When the sides of the arched roof spring from a circular or polygonal base, and meet in, or tend to, a point directly over the centre of that base.

SALON or SALOON, When a flat roof, or ceiling, is joined to the side walls by arcs of some one curve.

GROINS, When a vaulted roof is intersected by other vaults.

Of

Of vaulted roofs.

They are generally of one of these three sorts.

1\. *Circular*, } when the arch is some part
2\. *Elliptic*, }

of the periphery of { a circle. / an ellipse.

3\. *Gothic*, When the arch consists of two circular arcs meeting in a point directly over the middle of the breadth, or span, of the arch.

PROPOSITION XXXIII.

To find the solid content of circular, elliptic, or gothic, vaulted roofs.

RULE.

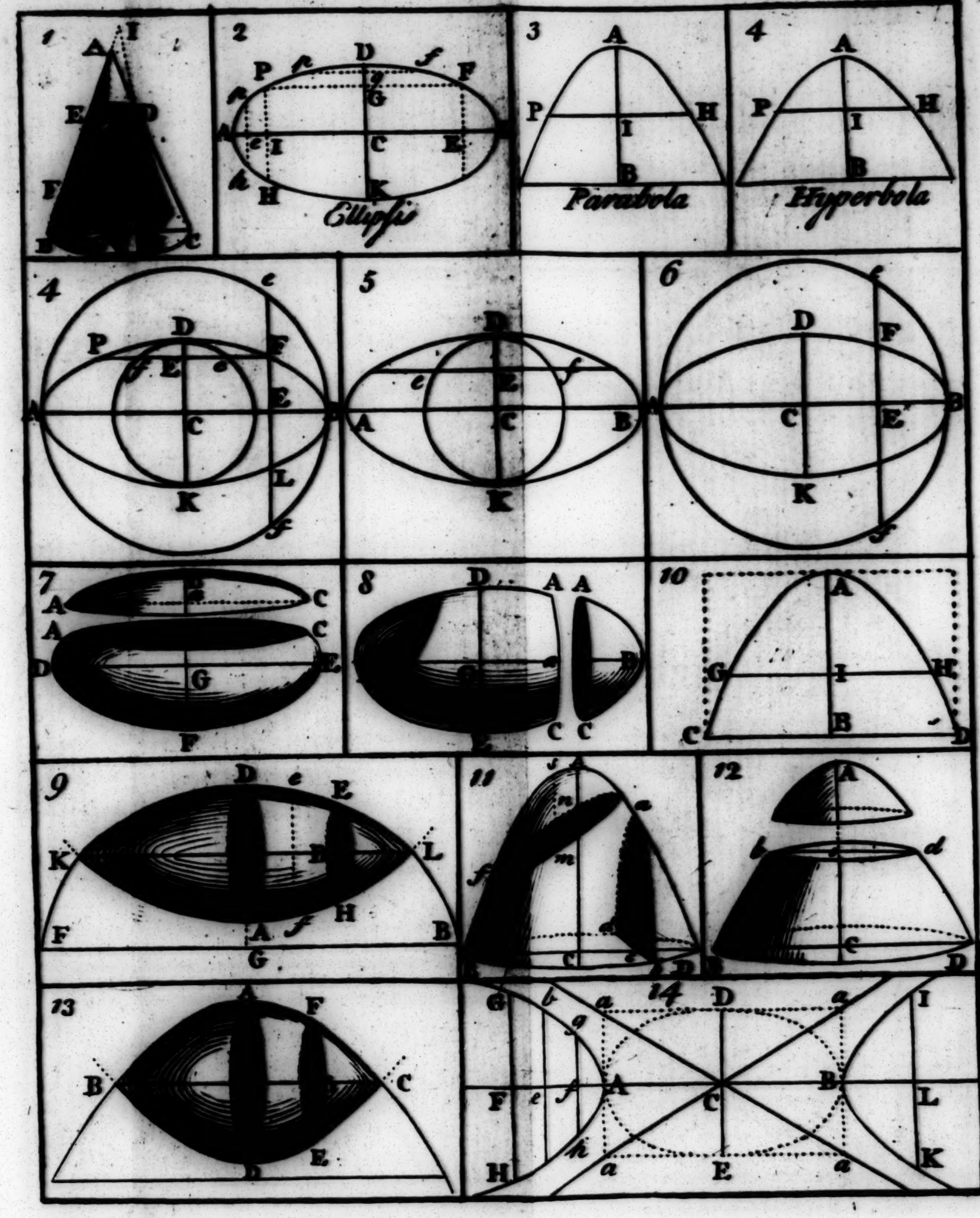
Multiply the area of one end by the length, and the product will be the solid content.

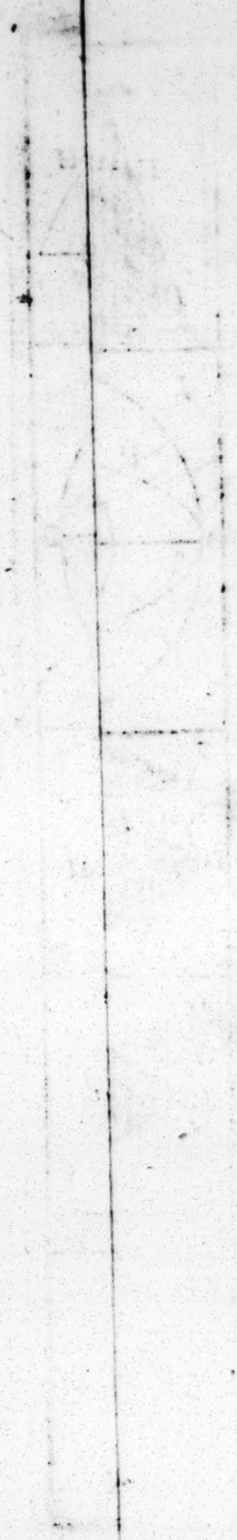
EXAMPLES.

I. *What is the solid content of a semicircular vault, whose span is 40 feet, and length 120 feet?*

Then $\left(3,1416 \times \overline{\frac{40}{2}}\right|^2 \times \frac{1}{2} \times 120 =\right)$ 75398,4 solid feet, is the content required.

II. *In*


Ellipsis
Parabola
Hyperbola



II. *In an elliptic vault whoſe ſpan is 40 feet, height 12, and length 80 feet: Required the ſolid content?*

Then ($40 \times 12 \times 0{,}7854 \times 80 =$) 30159,36 ſolid feet is the content ſought.

III. *What is the ſolid content of a gothic vault, whoſe ſpan is 48 feet, the chord of its arch, 48 feet, the diſtance of the arch from the middle of the chord is 18 feet, and the length of the vault is 18 feet?*

Now 1272,7874 is the area of the two circular ſegments, (ſee p. 163.)

And $\sqrt{48^2 - \overline{\frac{48}{2}}\,\big|^2} = 41{,}57$ is the height of the arch.

Then $\left(\overline{1272{,}7874 + 41{,}57 \times \frac{48}{2}} \times 60 =\right)$ 136228,044 is the ſolidity required.

If the ſolidity of the materials in either of theſe arches was required?

R U L E.

From the ſolid content including the arch, take the ſolid content of the void, and the remainder will be the ſolidity of the arch.

PROPOSITION XXXIV.

To find the concave, or convex surface of circular, elliptic, or gothic vaulted roofs.

RULE.

Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

NOTE.

If the convex surface of the vault is required; it will be most ready and accurate, to stretch a string over the convexity of the vault, and this string measured gives the length of the curve.

But for the concave surface, this method is not so applicable, and the length of the arch must be found, from proper dimensions, as shewn in Prop. XX. p. 158. or in Prop. II. p. 258.

Of

Of domes.

There may be a great variety of domes, arifing from the figure of their bafe, their height, and the nature of their curved fides: But as the moft common in ufe, are fuch whofe bafe is either a circle or a regular polygon, and whofe curved fides are circular, or elliptic quadrantal arcs, therefore thefe only will here be treated of.

PROPOSITION XXXV.

To find the folid content of a dome, whofe height and the dimenfions of its bafe are known.

RULE.

Multiply the area of the bafe by two thirds of the height, and the product will be the folid content.

Ex. I. *What is the folid content of a fpherical dome, the diameter of whofe circular bafe is 60 feet?*

Now $\overline{60}^2 \times 0{,}7854 = 2767{,}44$ is the area of the bafe.

And 30 is the height.

Then $\left(2767{,}44 \times \frac{2 \times 30}{3} =\right)$ 55348,8 cubic feet is the folid content.

Ex. II. *In a hexagonal spherical dome, one side of the base is 20 feet: Required the solid content?*

Now (by tab. I. p. 144. $0,8660254 \times 20 =$) 17,320508 is the radius of the inscribed circle, or height of the dome.

Also (by tab. I. p. 144. $2,5980762 \times \overline{20}|^2 =$) 1039,23048 is the area of the base.

Then ($1039,23048 \times 17,320508 \times \frac{2}{3} =$) 12000 solid feet is the content.

Ex. III. *A mason has built an octagonal elliptic dome, whose inside height is 60 feet; the diameter of its greatest inscrib'd circle is 40 feet; the thickness of the stone work at the bottom is 8 feet, and at the top is 4 feet: What will be the expence of this dome at* 12*s. a foot solid?*

Now $\left(\text{by tab. III. p. 145. } 3,3137084 \times \overline{\frac{40}{2}}\Big|^2 =\right)$ 1325,48336 is the area of the inner base.

And ($1325,48336 \times 60 \times \frac{2}{3} =$) 53019,3344 is the solid content of the void.

Also (by tab. III. p. 145. $3,3137084 \times \overline{28}^2 =$) 2597,9473856 is the area of the outward polygon.

And ($2597,9473856 \times 64 \times \frac{2}{3} =$) 110847,08845 is the solid content of the dome.

Then ($110847,08845 - 53019,3344 =$) 57827,75405 is the cubic feet of stone work, which will amount to 34696 £. 13 *s.* $0\frac{1}{2}$ *d.*

PRO-

PROPOSITION XXXVI.

To find the superficial contents of a spherical dome.

R U L E.

Twice the area of the base is the superficial contents required.

E X A M P L E.

What will the painting of a hexagonal spherical dome come to at 1 *s. a yard; each side of the base being* 20 *feet?*

Now (by tab. I. p. 144. $2{,}5980762 \times \overline{20}|^2 =$) 1039,23048 is the area of the base.

Then 2078,46096 is the superficial content.

And the expence will be about 103 £. 18 *s.* 6 *d.*

In elliptic domes, it will be near enough for practice, to work by the following,

R U L E.

To half the diameter at the base add the height, the sum multiplied by 1,5708 will give the superficial content nearly.

Examples to this are easily supplied.

Of Salons.

This sort of roofing or ceiling is generally used to cover such buildings or rooms, whose plan is either rectangular, circular, or a regular polygon: And the curved parts are circular or elliptic quadrantal arcs: Also, the sides of the flat part of the ceiling, are each alike equidistant from the walls of the room. In what follows, by flat ceiling, understand the middle or flat part of the Salon.

PROPOSITION XXXVII.

To find the solid content of a Salon, the figure and sides of the flat ceiling; the sides of the room, the height of the arch, and its projection from the wall being respectively known.

RULE.

Multiply the height of the arch, its projection, one fourth of the perimeter of the ceiling, and 3,1416 continually; call the product A.

From $\left\{\begin{matrix}\text{a side}\\\text{diam.}\end{matrix}\right\}$ of the room, take a $\left\{\begin{matrix}\text{like side}\\\text{diamet.}\end{matrix}\right\}$ of the ceiling; the square of the remainder multiplied by the proper factor; (page 144. 143.) the product multiplied by two thirds of the height of the arch, call B.

Multiply the area of the flat ceiling by the height of the arch, the product added to the sum of A and B, will give the solid content required.

Ex.

Ex. I. *What is the solid content of a Salon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide.*

Here the flat part of the ceiling is 16 by 12 feet.

Then $\left(2 \times 2 \times \frac{16 \times 2 + 12 \times 2}{4} \times 3,1416 =\right)$ $175,9296 = A$.

And $(\overline{20 - 16}|^2 \times 1,0000000 \times 2 \times \frac{2}{3} =)$ $21,3 = B$.

Therefore $16 \times 12 \times 2 + 175,9296 + 21,3 = 581,2629$ solid feet, is the content sought.

Ex. II. *A circular building of 40 feet diameter, and 25 feet high to the ceiling, is covered with a Salon, whose circular arch is 5 feet radius: Required the capacity of that room in cubic feet?*

Now $(40 - 5 \times 2 =)$ 30 is the diameter of the ceiling.

And $(30 \times 3,1416 =)$ 94,248 is the circumference thereof.

Then $\left(5 \times 5 \times \frac{94,248}{4} \times 3,1416 =\right)$ $1850,55948 = A$.

And $(\overline{40 - 30}|^2 \times 0,7854 \times 5 \times \frac{2}{3} =)$ $261,8 = B$.

Also $(\overline{30}|^2 \times 0,7854 \times 5 =)$ 3534,3 for the cylindric part of the Salon.

Hence the solid. of the Salon = 5646,65948.

And $(\overline{40}|^2 \times 0,7854 \times 20 =)$ 25132,8 is the solidity of the cylindric part of the room.

Then 30779,45948 is the solid content of that room.

Ex. III. *A gentleman who has in his garden a hexagonal ſummer-houſe, each ſide within being 7 feet, and the walls 18 inches thick; orders a maſon to cover this with a pyramidal ſtone roof, whoſe ſides ſhall riſe 6 inches within the outſide of the walls, and its height be equal to the radius of the circle inſcrib'd in the pyramid's baſe; alſo the inſide to be wrought in an elliptic ſalon, fitted to the ſpan of the room, the height of the arch to be 2 feet, and its projection 3 feet: But inſtead of the flat part of the ceiling, he will have a ſpherical hexagonal dome to ſpring from the upper extremities of the other arch: What will this roof come to, at 12 s. a foot ſolid?*

Now ($7 \times 0,8660254 =$) 6,0621778 is the radius of the circle inſcrib'd in the room. (tab. I. p. 144.)

Then 7,0621778 is the height of the pyramid; and alſo, is the radius of the circle inſcrib'd in its baſe.

The. $\left(\overline{7,0621778}|^2 \times {}^*3,4641016 \times \frac{7,0621778}{3} =\right)$ 406,710411 is the ſolidity of the pyramid.
(* Tab. III. p. 145.)

Again ($6,0621778 - 3 =$) 3,0621778 is the radius of a circle inſcrib'd in the flat ceiling of the ſalon.

Then ($3,0621778 \times {}^*1,1547005 =$) 3,5358981 is the ſide of that ceiling.
(* Tab. III. p. 145.)

And

And $\left(2 \times 3 \times \frac{3,5358981\ 6 \times 6}{4} \times 3,1416 =\right)$ 99,9753962 = A.

Alſo $(\overline{7 - 3,5358981}|^2 \times {}^*2,5980762 \times 2 \times \frac{2}{3} =)$ 41,569224 = B.
(* Tab. I. p. 144.)

Now $(\overline{3,0621778}|^2 \times {}^*3,4641016 =)$ 32,4826471 is the area of the flat part of the ceiling.
(* Tab. III. 145.)

Then (32,4826471 × 2 + A + B =) 206, 5098144 is the ſolid content of the ſalon.

But 3,0621778 the radius of the ceiling, is alſo the height of the dome.

And (32,4826471 × 3,0621778 × ₂ =) 66, 3117605 is the ſolid content of the dome.

Therefore 272,821575 is the ſolid content of the excavation or hollow of the roof.

And 133,888836 is the ſolidity of the ſtone work.

Hence the expence will be 80 £. 6 *s.* 8 *d.*

PROPOSITION XXXVIII.

To find the superficial content of a salon; the figure and sides of the flat ceiling, the sides of the room, the height of the arch, and its projection from the wall being respectively known.

RULE.

1st, Find the area of the flat part of the ceiling.

2d, Find the convex surface of a cylinder or cylindroid, whose length is equal to one fourth of the perimeter of the ceiling, and its diameters equal to twice the height and twice the projection of the arch.

3, Find the superficial content of a dome of the figure of the arch, and whose base is either a square, or a figure similar to that of the ceiling; the side being equal to the difference of a side of the room and a side of the ceiling.

The sum of these three articles will give the superficial content sought.

Note, In a {rectangular / circular / regul. polygonal} room, the base of the dome will be a {square / circle / like polygon.}

Examples to this prop. are easily supplied.

Of

Of Groins.

These arches or roofs may be conſidered, as ariſing from the interſections of ſegments of circular cylinders, or elliptical cylindroids, cut off by planes parallel to their axes.

There may be a great variety of Groins produced, but in this place no other will be conſidered, but thoſe whoſe interſections are at right angles; and of theſe, only ſuch as moſt commonly occur, and are formed.

I. By two circular, equal ſemicylinders; and called circular groins.

II. By two elliptical equal ſemicylindroids; either on the tranſverſe or conjugate axes; and called elliptical groins.

In either caſe, the groin arches ſpring over a ſquare baſe.

PROPOSITION XXXIX.

To find the solid content of the vacuity form'd by a groin arch; either circular or elliptical; the side of the square base, and the height of the groin being known.

RULE.

Multiply the area of the base by the height, the product multiplied by 0,9041295 will give the solid content required.

Ex. *What is the solid content of the vacuity form'd by a circular groin, one side of its square base being* 12 *feet?*

Now ($12 \times 12 =$) 144 is the area of the base.

And ($144 \times \frac{12}{2} =$) 864 is the product of the area of the base by the height.

Then ($864 \times 0,9041295 =$) 781,167888 is the solid content required.

Ex. II. *What is the solid content of the vacuity form'd by an elliptical groin; one side of its square base being* 20 *feet, and the height* 6 *feet?*

Then ($20 \times 20 \times 6 \times 0,9041295 =$) 2169,9108 is the solid content required.

PRO-

PROPOSITION XL.

To find the concave superficies of a circular groin arch; the side of the square base being known.

Rule. Multiply the area of the base by 1,1415923, and the product will give the superficies required.

This rule may be used for elliptical groins, the error being too small to be regarded in practice.

Ex. *What is the curve superficies of a circular groin arch; one side of its square base being 12 feet?*

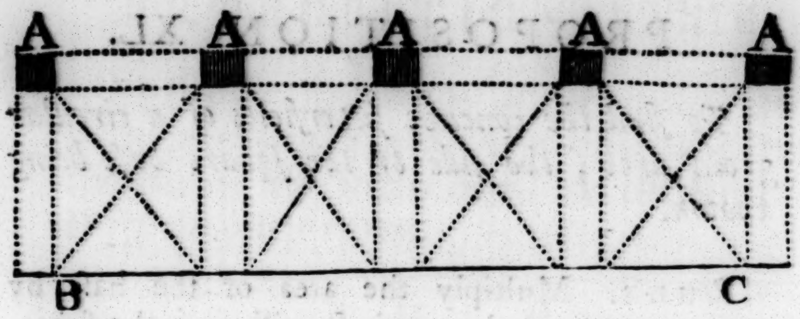
Then ($12 \times 12 \times 1,1415923 =$) 164,3892912 is the superficies required.

Either of these rules, may be applied, in practice, to groins contain'd under circular or elliptical segments, or on any rectangular base; the error, in some cases, being less than those, which frequently arise in taking the dimensions of the same work by different persons.

In measuring of work where there are many groins in a range, as a colonade, cloyster, piazza, &c.; the cylindrical pieces between the groins, and on their sides must be computed separately: And to find the solidity of the brick or stone-work which form the groin arches observe the following,

Rule. Multiply the area of the base, by the height, including the thickness of the work over the top of the groin, this product lessened by the solid content, found by the former rule, will leave the solidity of the work.

Ex.



Ex. *Let the figure above represent part of a groin'd piazza, joining to the wall* BC *of a house; where the piers* A *next the street, are each* 3 *feet broad,* 2 *feet thick, and* 7 *feet high; the span of the intermediate arches, are* 12 *feet each; the breadth of the piazza* 15 *feet in the clear; the height of the groins, above the piers,* 6 *feet: Now suppose the crown of the arch is* 2 *feet thick, and the upper surface made level, by working the spandrils up solid; What will the bill come to, at* 1 *s. a foot solid for the brick-work, and* 1 *s. a yard superficies for the stucco plaistering?*

Now ($12 \times 4 + 5 \times 3 =$) 63 feet is the length of the piazza.

($15 + 2 =$) 17 is the breadth.

($6 + 2 =$) is the height of the groin-work, including the thickness thereof over the crown.

And ($63 \times 17 \times 8 =$) 8568 is the solidity of the roof, including the solid content of all the vaults.

Also ($\overline{3 \times 2 \times 7} \times 5 =$) 210 is the solidity of the piers.

Then ($8568 + 210 =$) 8778 is the solidity of the work, including the deductions.

Again ($15 \times 12 \times 6 \times 0{,}9041295 \times 4 =$) 3905,83944 is the solid content of the vacuities form'd by the 4 groins.

And

And $\left(\frac{\overline{144 \times 0,7854}}{2} \times 2 \times 4 =\right)$ 452,3904 is the ſolid content of the curved vacuities between the piers in the front of the piazza.

Alſo $(\overline{15 \times 6 \times 0,7854 \times 3} \times 5 =)$ 1060,29 is the ſolid content of the curved vacuities between the piers and the houſe.

Then $(3905,83944 + 452,3904 + 1060,29 =)$ 5418,51984 is the ſolid content of all the deductions.

Therefore $(8778 - 5418,51984 =)$ 3359,48016 is the ſolid feet in all the work; which amounts to 167,974 £.

Again $(\overline{3 + 3 + 2 + 2} \times 7 \times 5 =)$ 350 is the ſuperficies of the five piers,

And $(\overline{63 \times 8} - \overline{0,7854 \times 12 \times 6} \times 4 =)$ 277,8048 is the ſurface of the front, above the piers.

Alſo $(\overline{15 \times 12, \times 1,1415923} \times 4 =)$ 821,946456 is the concave ſurface of the 4 groins.

And $(\overline{3,1416 \times 6 \times 2} \times 4 =)$ 150,7968 is the curve ſurface of the 4 arches between the piers.

Alſo $\left(\frac{\overline{15 \div 2 + 6}}{2} \times 3,1416 \times 3 \times 5 =\right)$ 318,087 is the curve ſurface of the 5 arches between the piers and the houſe.

The ſum of all theſe ſuperficies is 1918,635, &c. feet.

Or 213,181 yards, amounting to 10,659 £.

Therefore the whole expence will be 178 £. 12 *s.* 8 *d.*

SEC-

SECTION VII.

Of regular solids.

A Sphere is said to circumscribe a solid, when the angular points touch the concave surface of that sphere.

A Sphere is said to be inscrib'd in a solid, when the plane sides touch the convex surface of that sphere.

A regular solid, is a body contain'd under equal, regular and like planes; alike posited, and equally distant from the centre.

There are only five regular solids.

I. The Tetraedron, contain'd under four equal and equilateral plane triangles; forming four solid angles, each of three triangles; and having six linear edges, and twelve plane angles.

II. The Hexaedron, contain'd under six equal squares; forming eight solid angles, each of three squares; and having twelve linear edges, and twenty-four plane angles.

III. The OCTAEDRON, contain'd under eight equal and equilateral plane triangles; forming six solid angles, each of four triangles; having twelve linear edges, and twenty-four plane angles.

IV. The DODECAEDRON, contain'd under twelve equal, equilateral, and equiangular pentagons; forming twenty solid angles, each of three pentagons, and having thirty linear edges and sixty plane angles.

V. The ICOSAEDRON, contain'd under twenty equal and equilateral triangles; forming twelve solid angles, each of five triangles; and having thirty linear edges, and sixty plane angles.

By the following table, the superficies, solidities, radii of the circumscrib'd and inscrib'd spheres, and the linear sides or edges, of any of the regular solids, may be readily computed.

Let $\left\{\begin{matrix} S \\ Z \\ X \end{matrix}\right\}$ repreſent the $\left\{\begin{matrix} \text{linear ſide} \\ \text{ſuperficies} \\ \text{ſolidity} \end{matrix}\right\}$ of a regular ſolid.

When	Then	Tetraedron.	Hexaedron.	
S = 1	R =	0,6123724	0,8660254	1
	r =	0,2041241	0,5000000	2
	Z =	1,7320508	6,0000000	3
	X =	0,1178511	1,0000000	4
R = 1	S =	1,6329932	1,1547006	5
	r =	0,3......	0,5773503	6
	Z =	4,6188023	8,......	7
	X =	0,5132002	1,5396006	8
r = 1	S =	4,8989795	2,......	9
	R =	3,.......	1,7320508	10
	Z =	41,5692192	24,......	11
	X =	13,8564064	8,......	12
Z = 1	S =	0,7598357	0,4082483	13
	R =	0,4653025	0,3535534	14
	r =	0,1551008	0,2041241	15
	X =	0,0517003	0,0680413	16
X = 1	S =	2,0395489	1,.......	17
	R =	1,1547006	0,8660254	18
	r =	0,4163417	0,5......	19
	Z =	7,2056240	6,.......	20

And $\left\{ \begin{matrix} R \\ r \end{matrix} \right\}$ the radius of the $\left\{ \begin{matrix} \text{circumſc.} \\ \text{inſcrib'd} \end{matrix} \right\}$ ſphere.

	Octaedron.	Dodecaedron.	Iſocaedron.
1	0,7071068	1,4012585	0,9510565
2	0,4082483	1,1135164	0,7557613
3	3,4641016	20,6457288	8,6602540
4	0,4714045	7,6631188	2,1816951
5	1,4142136	0,7136442	1,0514622
6	0,5773503	0,7946545	0,7946545
7	6,9282032	10,5140223	9,5745413
8	1,3	2,7851639	2,5361507
9	2,4494897	0,8980560	1,3231691
10	1,7320508	1,2584086	1,2584086
11	20,7846096	16,6508731	15,1621684
12	6,9282032	5,5502910	5,0540561
13	0,5372850	0,2200822	0,3398088
14	0,3799178	0,3083920	0,3231774
15	0,2193457	0,2450651	0,2568144
16	0,0731152	0,08168837	0,08560479
17	1,2848990	0,5072221	0,7710254
18	0,9080604	0,7107492	0,7332887
19	0,5245576	0,5648000	0,5827111
20	5,7191069	5,3116140	5,1483486

The

The uſe of the foregoing table will be ſufficiently illuſtrated by the ſolution of the following Caſes; wherein, to avoid repetitions: By a ſolidity, a ſuperficies, or a ſide, is meant the ſolidity, the ſuperficies, or, the linear ſide or edge of a regular ſolid: And, by a radius, is meant, the radius of the ſphere that can either circumſcribe, or be juſt contain'd in, that regular ſolid.

Let N repreſent the tabular number; correſponding to either ſolid, and, its ſide; its radius; its ſuperficies; its ſolidity; where S, R, *r*, Z or X, = 1.

CASE I.

A $\left\{\begin{matrix}\textit{ſide}\\ \textit{radius}\end{matrix}\right\}$ *being given; to find a* $\left\{\begin{matrix}\textit{radius}\\ \textit{ſide.}\end{matrix}\right.$

RULE.

Multiply the given $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ by N $\left\{\begin{matrix}\text{radius, where } S = 1;\\ \text{ſide, where } R, \text{ or, } r = 1\end{matrix}\right\}$ and the product will be the $\left\{\begin{matrix}\text{radius}\\ \text{ſide}\end{matrix}\right\}$ required.

EX-

EXAMPLE I.

If the side of a dodecaedron is 2; required the radii of the circumscrib'd and inscrib'd spheres?

Under the name dodecaedron, and against R and r, where S = 1; are the numbers 1,4012585 and 1,1135164.

$$\text{Then}\left.\begin{array}{l}(1{,}4012585\times 2=)\,2{,}8025170=R\\(1{,}1135164\times 2=)\,2{,}2270328=r\end{array}\right\}\text{required.}$$

EXAMPLE II.

If the radius of a sphere, circumscribing a dodecaedron, be 2,802517; required the side of that dodecaedron; and the radius of its inscrib'd sphere?

Against $\left\{\begin{array}{c}S\\r\end{array}\right\}$ where R = 1; and under dodecaedron,

is $\left\{\begin{array}{l}0{,}7136442\\0{,}7946545\end{array}\right\}$ which multiplied by 2,802517;

gives $\left\{\begin{array}{l}2\text{ - - - - - }=S\\2{,}2270326=r\end{array}\right\}$ required.

EXAMPLE III.

If the radius of a ſphere, inſcrib'd in a dodecaedron, is 2,2270328: Required the ſide of that dodecaedron, and the radius of the circumſcrib'd ſphere?

Againſt $\left\{\begin{matrix} S \\ R \end{matrix}\right\}$ where $r = 1$; and under dodecaedron, is $\left\{\begin{matrix} 0{,}8980560 \\ 1{,}2584086 \end{matrix}\right\}$ which multiplied by 2,2270328; gives $\left\{\begin{matrix} 2 \text{ - - - - - } = S \\ 2{,}8025171 = R \end{matrix}\right\}$ required.

CASE II.

A $\left\{\begin{matrix} \textit{ſide} \\ \textit{radius} \end{matrix}\right\}$ *being given; to find a ſuperficies.*

RULE.

Multiply the ſquare of the given $\left\{\begin{matrix} \text{ſide} \\ \text{radius} \end{matrix}\right\}$ by N ſuperficies $\left\{\begin{matrix} \text{where } S \text{ - - - - } = 1. \\ \text{where } R\text{, or, } r = 1. \end{matrix}\right\}$ And the product will give the ſuperficies required.]

EXAMPLE I.

What is the ſuperficies of a dodecaedron whoſe ſide is 2?

Then $(20{,}6457288 \times \overline{2}|^2 =)$ $82{,}5829156 = Z$ required.

EX-

EXAMPLE II.

What is the superficies of a dodecaedron, [illegible] *sphere whose radius is 2,802517?*

Then $(10{,}5146223 \times \overline{2{,}802517}|^2 =)\,82{,}5829159$ $=$ Z required.

EXAMPLE III.

What is the superficies of a dodecaedron, [illegible] *sphere, whose radius is 2,2270328?*

Then $(16{,}6508731 \times \overline{2{,}2270328}|^2 =)\,82{,}5829156$ $=$ Z required.

CASE III.

A $\left\{\begin{array}{c}\textit{side}\\ \textit{radius}\end{array}\right\}$ *being given; to find a solidity?*

RULE.

Multiply the cube of the given $\left\{\begin{array}{c}\text{side}\\ \text{radius}\end{array}\right\}$ by N solidity $\left\{\begin{array}{c}\text{where } S = 1\\ \text{where } R, \text{ or, } r = 1\end{array}\right\}$ and the product will give the solidity required.

EXAMPLE I.

What is the solidity of a dodecaedron whose side is 2?

Then $(7{,}6631188 \times \overline{2}|^3 =)$ $61{,}3049504 =$ X required.

EXAMPLE II.

What is the solidity of a dodecaedron inscrib'd in a sphere whose radius is 2,80257?

Then $(2{,}7851639 \times \overline{2{,}80257}|^3 =)$ $61{,}3049499$ $=$ X required.

EX-

EXAMPLE III.

What is the solidity of a dodecaedron, circumscribing a sphere, whose radius is 2,2270328?

Then $(5,5502910 \times \overline{2,2270328}|^3 =) 61,3049509 = X$ required.

CASE IV.

A superficies being given; to find $\left\{ \begin{matrix} \textit{side} \\ \textit{radius.} \end{matrix} \right.$

RULE.

Multiply the square root of the given superficies, by N $\left\{ \begin{matrix} \text{side} \\ \text{radius} \end{matrix} \right\}$ where $Z = 1$; and the product will be the $\left\{ \begin{matrix} \text{side} \\ \text{radius} \end{matrix} \right\}$ required.

EXAMPLE.

If the superficies of a dodecaedron is 82,5829152; *required S, R, r?*

I. $(0,2200822 \times \sqrt{82,5829152} =) 2,\text{-----} = S.$

II. $(0,3085920 \times \sqrt{82,5829152} =) 2,8025166 = R.$

III. $(0,2450651 \times \sqrt{82,5829152} =) 2,2270324 = r.$

CASE V.

A solidity being given; to find a $\left\{ \begin{matrix} \textit{side} \\ \textit{radius.} \end{matrix} \right.$

RULE.

Multiply the cube root of the given solidity by the N $\left\{ \begin{matrix} \text{side} \\ \text{radius} \end{matrix} \right\}$ where $X = 1$; and the product will be the $\left\{ \begin{matrix} \text{side} \\ \text{radius} \end{matrix} \right\}$ required.

EX-

EXAMPLE.

If the ſolidity of a dodecaedron is 61,3049504; *Required S, R, r?*

I. $(0,5072221 \times \sqrt[3]{61,3049504} =)$ 2, $= S$.

II. $(0,7107492 \times \sqrt[3]{61,3049504} =)$ 2,8025168 $= R$.

III. $(0,5648000 \times \sqrt[3]{61,3049504} =)$ 2,2270323 $= r$.

CASE VI.

A ſuperficies being given; to find a ſolidity.

RULE.

Multiply the given ſuperficies by its ſquare root; the product multiplied by the N ſolidity, where $Z = 1$, will give the ſolidity required.

EXAMPLE.

What is the ſolidity of a dodecaedron, whoſe ſuperficies is 82,5829152?

Now $\sqrt{82,5829152} = 9,0875142$.

And $82,5829152 \times 9,0875142 = 750,4734144$.

Then $750,4734144 \times 0,08168837 = 61,3049499 = X$.

CASE VII.

A ſolidity being given; to find a ſuperficies.

RULE.

Divide the given ſolidity by its cube root; the quotient multiplied by N ſuperficies, where $X = 1$, will give the ſuperficies required.

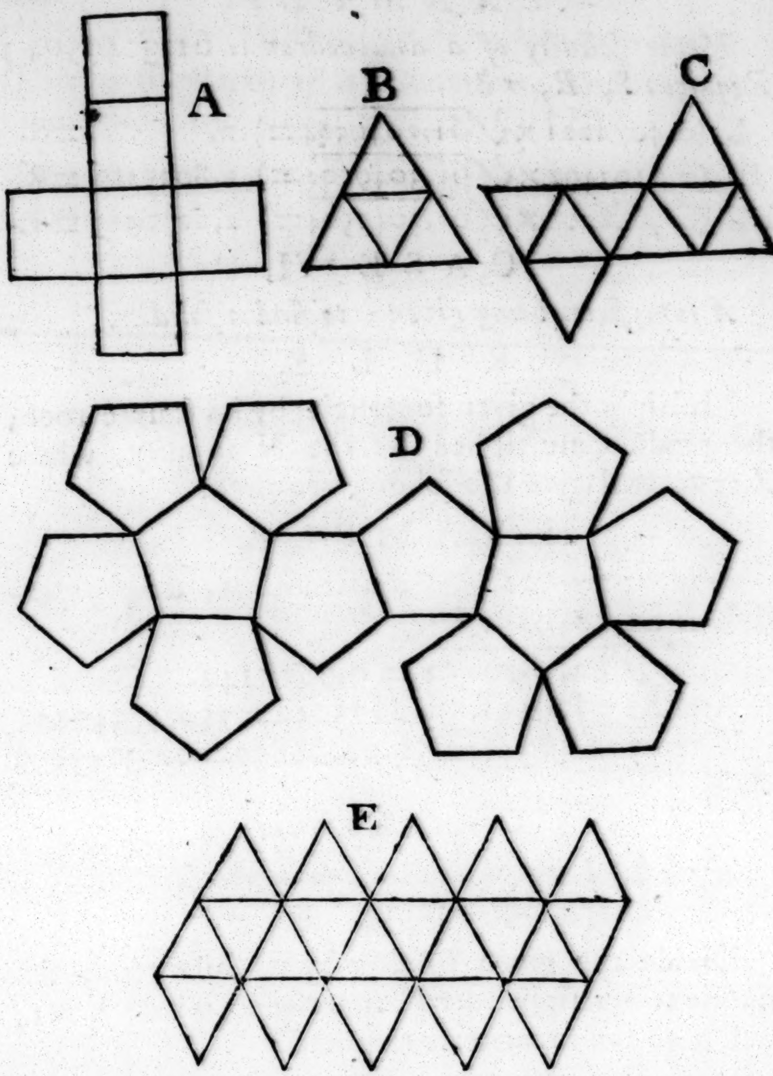
EXAMPLE.

What is the ſuperficies of a dodecaedron whoſe ſolidity is 61,3049504?

Now $\sqrt[3]{61,3049504} = 3,943046$.

And $\dfrac{61,3049504}{3,943046} = 15,5476122$.

Then $15,5476122 \times 5,3116140 =)$ $82,5829146 = Z$.



In five ſuch figures as theſe, made of paſte-board, or any other pliable ſubſtance; if the lines be cut half through, and the parts turned up and glued together, the figures will repreſent the five regular ſolids: *viz.* A, the hexaëdron; B, the tetraëdron; C, the octaëdron; D, the dodecaëdron; and E, the Icoſaëdron.

SECTION VIII.

Of ſpecific gravity.

THE ſpecific gravity of a body, is the relation that the weight of a magnitude of that kind of body, has to the weight of an equal magnitude of another kind of body.

In this compariſon of the weights of bodies, it is convenient to conſider one body as the ſtandard or unit, to which the others are to be compared: and as rain water is nearly alike in all places; therefore, this ſeems to be the moſt convenient for a ſtandard.

It has been found by repeated experiments, that a cubic foot of rain water, weigh'd $62\frac{1}{2}$ pounds averdupoiſe; conſequently $\left(\frac{62,5}{1728}\right)$0,0361689 8 ℔ is the weight of one cubic inch of rain water.

The knowledge of the ſpecific gravities of bodies, is of great uſe in computing the weights of ſuch bodies, as are too heavy or too unweildy to have their weight diſcover'd by other means.

A TABLE *shewing the specific gravity, to*
And the weight of a cubic inch of each,

Bodies.	Sp. gravity.	Wt. ℔ averd.
Fine gold - - - - -	19,640	0,7103587
Eng. gold coin - -	19,520	0,7060185
Coast gold - - - -	18,888	0,6828703
Quickfilver - - -	13,762	0,4976574
Lead - - - - - - - -	11,313	0,4091696
Fine filver - - - -	11,091	0,4011501
Eng. filver coin -	10,629	0,3844400
Cast filver - - - -	10,528	0,3807870
Copper - - - - - -	8,769	0,3171658
Cast brass - - - -	8,104	0,2929832
Steel - - - - - - - -	7,850	0,2839265
Bar iron - - - - - -	7,764	0,2808159
Block tin - - - - -	7,238	0,2617901
Cast iron - - - - -	7,135	0,2580647
Loadstone - - - -	5,106	0,1846788
Blue flate - - - - -	3,500	0,1264914
Vein'd marble - -	2,702	0,0977286
Common glass - -	2,600	0,0940393
Flintstone - - - -	2,582	0,0933883
Portlandstone - -	2,570	0,0929543
Freestone - - - - -	2,352	0,0915788
Brick - - - - - - - -	2,000	0,0723379
Ivory } Horn } - - - - - -	1,832	0,0662606

Note, 7000 grains make 1 ℔ averdupoise.

And 5760 grains make 1 ℔ troy.

Therefore, the ℔ averd. : ℔ troy : : 700 : 576.

rain-

rain water; of metals and other bodies: in parts of a pound averdupoise.

Bodies.	Sp. gravity.	Wt. ℔ averd.
Brimftone - - - -	1,800	0,0651042
Clay - - - - - - - -	1,712	0,0619213
Lignum Vitæ - -	1,327	0,0479862
Coal - - - - - - - -	1,255	0,0453921
Pitch - - - - - - - -	1,150	0,0415943
Mahogany wood	1,063	0,0384475
Dry box wood - -	1,030	0,0372530
Milk - - - } Sea water } - -	1,030	0,0372530
Rain water - - - -	1,000	0,0361690
Bees wax - - - -	0,995	0,0359881
Dry oak - - - - -	0,915	0,0330946
Olive oil - - - - -	0,913	0,0330222
Beech - - - - - - -	0,854	0,0308883
Dry elm } Dry afh } - - - -	0,800	0,0289352
Dry wainfcot - -	0,747	0,0270182
Dry yellow fir - -	0,657	0,0237630
Cedar - - - - - - -	0,613	0,0221715
Dry white deal - -	0,569	0,0205801
Cork - - - - - - - -	0,240	0,0186805
Air - - - - - - - - -	0,0012	0,0000434

Confequently ℔ averd. mult. by $\frac{5,76}{7}$ gives ℔ troy.

And ℔ troy multi. by $\frac{700}{576}$ gives ℔ averd.

Note, $\frac{700}{576} = 1,215278$ nearly.

CASE I. *The linear dimensions, or solidity, of a body being given; to find its weight.*

RULE. Multiply the cubic inches contain'd in that body, by the tabular weight corresponding to the name of the same kind; and the product will give the weight in pounds averdupoise.

EX. I. *What is the weight of a piece of oak, of a rectangular form, whose length is 56 inches; breadth 18, and depth 12 inches?*

Now ($56 \times 18 \times 12 =$) 12096 inches is the solidity.

Then ($12096 \times 0{,}0330946 =$) 400,3122816 lb is the weight required.

EX. II. *What is the weight of an iron shot, of 7 inches diameter?*

Now ($\overline{(7|}^{3} \times 0{,}5206 =$) 179,5948 inches is the solidity.

Then ($179{,}5948 \times 0{,}2580647 =$) 46,34706 lb is the weight required.

Ex. III. *What is the weight of an iron bomb-shell, of 3 inches thick; the greatest diameter being 16 inches?*

Now $(16 - \overline{3 \times 2} =)$ 10 is the diameter of the concavity.

And $(\overline{16}|^3 \times 0{,}5236 = 2144{,}6656$.

Also $(\overline{10}|^3 \times 0{,}5236 = 523{,}6$.

Then $(2144{,}6656 - 523{,}6 =)$ 1621,0656 is the solidity of the shell.

Therefore $(1621{,}0656 \times 0{,}2580647 =)$ 418, 3398 lb is the weight required.

Ex. IV. *Required the weight of one of the Portland key-stones, to the middle arch of* Westminster-*bridge: The diameter of the arch being* 76 *feet; the height of the key-stone* 5 *feet; the chord of its greatest breadth, to the front of the arch,* 3 *feet* 4 *inches; and its depth, in the arch,* 4 *feet?*

Now $(76 + 5 : 3{,}\dot{3} :: 76 :)$ 3,127572 is the chord of the least breadth of the key-stone.

Here the chords and their arcs may be supposed equal; the excess, in the greatest arc, not being more than about $\frac{1\frac{1}{2}}{5000}$ part of an inch.

Now $(\overline{3{,}127572 \times 4 + 3{,}\dot{3} \times 4} \times \frac{2}{3} \times \frac{5}{3} =)$ 64, 60905 feet is the solidity of the key-stone.

Then$(64{,}60905 \times 1728 \times 0{,}0929543 =)$ 10377, 83062 lb.

Or 4 tons, 12 hund. 2 quart. 17,83 lb is the weight required.

Case II. *The weight of a body being given; to find the solidity.*

Rule. Divide the given weight, in pounds averdupoise, by the tabular weight, corresponding to the name of the same kind; and the quotient will be the solidity in cubic inches.

Ex. I. *What will a block of marble, weighing 8 tons, 14 C. wt. come to, at 6 s. a foot solid?*

Now 8 t. 14 c. = 19488 ℔.

Then $\left(\frac{19488}{0,0977286} \div 1728 =\right)$ 115,4 are the cubic feet.

And $(115,4 \times 0,3 £. =)$ 34 £. 12 *s.* 5 *d.* is the cost.

Ex. II. *What is the diameter of an iron shot, weighing 42 pounds averd.?*

Now $\left(\frac{42}{0,2580647} =\right)$ 162,7499 are the cubic inches.

Then $\left(\sqrt[3]{\frac{162,7499}{0,5236}} =\right)$ 6,7743 is the diameter required.

How

How many inches will a cubic foot of dry elm sink in common water?

ANSWER. It will sink until a bulk of water, equal to the part immersed, be equal in weight to that of all the elm.

Now ($1728 \times 0{,}0289352 =$)50,0000256 lb is the weight of a foot of elm; and also the weight of the water displaced.

And $\left(\frac{50{,}0000256}{0{,}036169} =\right)$ 1382,4 are the cubic inches immersed.

Then $\left(\frac{1382{,}4}{144} =\right)$ 9,6 inches, is the depth to which a cubic foot of elm will sink in common water.

How much weight is just necessary to immerse a cubic foot of yellow fir in sea water?

ANSWER. So much weight as is equal to the difference between the weights of a cubic foot of sea water and that of fir.

Now ($0{,}037253 \times 1728 =$) 64,373184 lb is the weight of a cubic foot of sea water.

And ($0{,}023763 \times 1728 =$) 41,062464 lb is the weight of a cubic foot of dry fir.

Then ($64{,}373184 - 41{,}062464 =$) 23,31072 lb must be added to a cubic foot of fir to immerse it in sea water.

OR. *The difference between the specific weights, multiplied by the cubic inches in the body to be immersed, will give the additional weight.*

Thus ($0{,}037253 - 0{,}023763 =$) $0{,}01349 \times 1728 =$ 23,31072 lb as found before.

How many ſolid feet of yellow fir, laſh'd to a braſs cannon of 56 C. wt. will be ſufficient to keep it afloat at ſea.

Now 56 C. wt. = 6272 ℔ averdupoiſe.

And $\left(\frac{6272}{0,2929832} =\right)$ 21047,371 are the cubic inches of braſs.

Hence $(21047,371 \times 0,037253 =)$ 797,4887 ℔ is the weight of a bulk of ſea water, equal to that of the cannon.

Therefore $(6272 - 797,4887 =)$ 5474,5113 ℔ of the cannon is to be buoy'd up by the fir.

And the weight to be buoy'd up, divided by the difference of the ſpecific weights, of the body which is to buoy, and the fluid in which it is to float, will give the ſolidity of the buoying body.

That is, $\left(\frac{5474,5113}{0,037253 - 0,023763} =\right)$ 405819,936 cubic inches of fir will ſuſtain the cannon.

But 405819,936 cubic inches = 234,8495 feet ſolid.

Conſequently 12 pieces of fir timber, each of about a foot ſquare and 20 feet long, will ſuffice to keep ſuch a piece of cannon afloat in ſea water.

How

How thick must be the metal of a concave copper-ball, 6 inches in its outside diameter, so as to sink to its centre in common water?

Now ($\overline{6}|^3 \times 0{,}5236 =$) 113,697 cubic inches, is the solidity of that sphere.

And $\left(\frac{113{,}0976}{2} = \right)$ 56,5488 cubic inches to be immersed.

Or cubic inches of water to be remov'd.

Therefore ($56{,}5488 \times 0{,}036169 =$) 2,0453 ℔ is the weight of the water displaced, or the weight of the copper ball.

Consequently $\left(\frac{2{,}0453}{0{,}3171658} = \right)$ 6,44867 are the cubic inches of copper in that ball.

But ($\overline{6}|^2 \times 0{,}7854 \times 4 =$) 28,2744 square inches, is the superficies of the ball.

Then $\left(\frac{6{,}44867}{28{,}2744} = \right)$ 0,2881 part of a linear inch, is the thickness (nearly) required.

SCHO-

SCHOLIUM.

The ſolidity, or weight, of any body, however irregular, may be very exactly determin'd, as follows.

Into an uniform veſſel, (whoſe horizontal ſections may be readily computed,) pour ſo much water, as may be judg'd neceſſary to cover the body (whoſe ſolidity is required) when immers'd therein; and note the height of the fluid in the veſſel: Immerſe the body, and note how high the fluid has riſen: Then the ſolid content of the additional ſpace occupied by the fluid, on account of the immerſed body, will be equal to the ſolidity of that body: And conſequently its weight is readily known.

And thus, may the ſolidity of ſtatues, &c. be very exactly computed.

The following remarks are here added, becaufe they may be ufeful to fome perfons.

I. Of *Newcaftle* coal, 60 folid feet are equal to 1 chaldron.

Therefore; divide the folid content in feet, of a vault, cellar, or other place by 60, and the quotient will fhew how many chaldrons of coals that place will hold.

II. The contents of a *Winchefter* bufhel when heaped, is in proportion to the contents of the fame bufhel when ftruck, as 4 to 3.

Therefore the conical heap is one third of the cylindrical contents.

III. Straw is generally fold in truffes, each of 36 ℔ averdupoife, and 36 truffes make a load.

IV. Hay is generally fold by the load, containing 36 truffes, each of 56 ℔, or half a hundred weight; fo that a load of hay weighs 18 hundred weight.

Truffes of hay cut out of a rick, are of various dimenfions, according to the goodnefs of the hay, and the care taken in laying it up.

The

The largest trusses will seldom exceed 14 solid feet; and the smallest will rarely give less than 7 feet solid, and yet each of them weigh half a hundred weight.

But upon a mean, a truss may be reckoned at 10 or 11 feet solid; and a load will contain about 360 or 400 feet.

In ricks of good hay, the farmers commonly cut their trusses of five spans in length, three and a half spans in breadth, and one and a half, or two spans in thickness; or try a thickness that will, with the other dimensions, make half a hundred weight.

Hay-stacks stand either on a rectangular, or on a circular base; and are composed of two tapering solids; the lower one, with its least end downwards; and the higher one, with its least end uppermost; so that the thickest part of the rick is the end common to both solids: The height of the lower solid is commonly from about 6 to 10 feet; and the height of the upper one, from about 24 to 18 feet.

To

To find the number of loads contain'd in a circular hay-rick.

RULE.

1. Multiply twelve times the height of the lower solid, by the square of half the sum of its greatest and least diameters.

2. Multiply five times the height of the upper solid, by the square of its greatest diameter.

3. The sum of these two products, multiplied by 654; cut off the four right-hand figures, and those to the left-hand will be solid feet.

4. The solid feet divided by 400, or by 360, will give the loads.

EXAMPLE.

How many loads are contained in a round hay-rick, whose lower solid is 8 feet high, 24 feet in the lesser diameter, and 30 feet in the greater diameter; and the upper solid is 20 feet high?

Add { 30	Mul. { 30	Mul. { 159984
24	30	654
2) 54	Mul. { 900	639936
Mul. { 27	100	799920
27	Add { 90000	959904
189	69984	4,00) 10462,9536 so. fe.
54	159984	26,$\frac{1}{6}$ loads nearly.
729		
96		
4374		
6561		
69984		

To

To find the loads contained in a rectangular hay-rick.

RULE.

1. In the lower solid; multiply the lesser length by the lesser breadth; and the greater length by the greater breadth; add the products together, multiply the sum by half the height, reserve the product.

2. In the upper solid; multiply the length of the greater end by its breadth; divide the product by twelve; multiply the quotient by seven times the height; reserve the product.

3. The sum of these two reserved products will give the number of solid feet; which divided by 360 or 400, will give the loads.

EXAMPLE.

In a rectangular rick, the lower solid is 10 *feet high,* 36 *feet long, and* 12 *feet wide at the lesser end,* 40 *feet long and* 18 *feet wide at the greater end; and the upper solid* 24 *feet high: What is the rick worth at* 30 *shillings a load?*

Mul. $\left\{\begin{array}{r}36\\12\end{array}\right.$ Mul. $\left\{\begin{array}{r}18\\40\end{array}\right.$ Mul. $\left\{\begin{array}{r}18\\40\end{array}\right.$

$\begin{array}{lrl} & \underline{} & \\ \text{Add} & 432 & \\ & \underline{720} & \\ & 1152 & \\ & \underline{5} & \\ \text{Add} & 5750 & \text{1st prod.} \\ & \underline{10080} & \\ 4{,}00(& 15830 & \\ & 39\tfrac{1}{2} & \text{loads.} \end{array}$ $\quad \begin{array}{r} \underline{} \\ 720 \end{array} \quad \begin{array}{lrl} & \underline{} & \\ 12) & \underline{720} & \\ \text{Mul.} & 60 & \\ & \underline{7} & \\ \text{Mul.} & 420 & \\ & \underline{24} & \\ & 1680 & \\ & \underline{840} & \\ & 10080 & \text{2d produ.} \end{array}$

Therefore the rick is worth 59 £. 5 s.

The reader must not expect by these rules to compute the contents of hay-stacks, as accurately as the other solids in this work are computed by their respective rules; but he will by these means be able to value the stock of hay in ricks as nearly, perhaps, as by any other method; if regular methods for these things be already extant.

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